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Journal
of
The Franklin Institute
Devoted to Science and the Mechanic Arts

Vol. 209 FEBRUARY, 1930 No. 2

THE VARIATION OF THE RESIDUAL IONIZATION WITH PRESSURE AT DIFFERENT ALTITUDES, AND ITS RELATION TO THE COSMIC RADIATION.

BY

W. F. G. SWANN, D.Sc.,

Director of the Bartol Research Foundation.
Member of the Institute.

BARTOL RESEARCH FOUNDATION.
Communication No. 44.

EXPERIMENTS on the variation of residual ionization in a closed vessel have been made by several investigators, some of the most recent work having been performed by K. M. Downey,[1] H. F. Fruth,[2] and J. W. Broxon,[3] under the writer's direction. One of the most characteristic features of the results of these investigations was the conclusion that in the iron vessel used, which was about a foot in diameter, the increase of rate of production of ions per atmosphere increase decreased with increase of pressure, until, in the neighborhood of a hundred atmospheres, it had attained a value very small compared with its value at pressures comparable with atmospheric pressure. In view of these considerations, and of the more recent work of Prof. Millikan and his collaborators on the cosmic radiation at various altitudes, it seemed desirable to

[1] "Variation with Pressure of the Residual Ionization due to Penetrating Radiation," *Physical Review*, November 1920. "The Variation of the Residual Ionization in Air with Pressure, for a Range of 57 Atmospheres," *Physical Review*, August 1922.

[2] "Variation with Pressure of the Residual Ionization of Gases," *Physical Review*, August 1923.

[3] "Natural Ionization in Gases," *Physical Review*, Vol. 27. 1925.

extend the pressure measurements to high altitudes. It was with this object in view that the experiments to be described were undertaken under the auspices of the Bartol Research Foundation, in the fall of 1926. Although the writer has published a preliminary report upon this work,[4] the full account of it has been delayed mainly because of the large amount of work involved in taking into account the various types of corrections involved, such as the absorption of radiation by the buildings, etc., and on account of the writer's desire to develop a type of analysis of the results which seemed best calculated to exhibit their real content.

Experiments were carried out on the top of Pike's Peak at an altitude of 14,000 feet; at Colorado Springs, at an altitude of 6,000 feet; and subsequently at sea level at Yale University, New Haven, so that data for these elevations were available for the interpretation of results.

The iron sphere used was the same as had been used by Broxon, Fruth and Miss Downey in the earlier experiments. It had a volume of 14,800 ccs., and a wall thickness of about 2.5 cms. The compressed air was obtained from high pressure cylinders which were filled by the Paschall Oxygen Company in Philadelphia, and allowed to age for a month before use, so as to permit of the decay of the radium emanation in the air.[5] In order to shield the apparatus from the softer radiations, about a ton of lead was used, cast in the form of rings, with an annular thickness of 2 inches, and the ends of the lead cylinder so formed were closed at the top and bottom by means of lead to a total thickness of two inches in each case. The total weight of the apparatus amounted to about 2 tons. Fortunately, there is a very good automobile road leading to the top of Pike's Peak, so that no serious difficulties were experienced in transportation.

In addition to a cafeteria for the convenience of tourists,

[4] JOURNAL FRANKLIN INSTITUTE, Vol. 205, No. 6, June 1928. *Physical Review*, Vol. 29, Feb. 1927.

[5] Normally, atmospheric air contains about 130×10^{-12} curie of radium emanation per cubic meter. This would produce, on an average, about 5 ions per cc. per second. After the radium emanation had decayed for a month, the rate of production of ions would be only about 0.022 ions per cc. per sec., so that the increase of ions per cc. per sec., per atmospheric increase would be less than this amount.

there is, at the summit, a sturdy one-story building with concrete walls about 3 feet thick, but with a roof of light wood construction. The building, which is used as a hotel in the summer time, was made available for my use during the fall through the courtesy of the owners, Messrs. Stewart Bros. of Colorado Springs. The experiments were performed in this building. An idealistic attitude would have favored mounting the sphere in the open with, at most, a thin tent over it. Such a procedure was deemed impracticable; for, the winds which arise suddenly on the summit of the mountain would blow any light structure away very quickly. Then, the necessity of handling the heavy parts of the apparatus, practically single handed, with only two-thirds of an atmosphere of pressure to breathe, presented a problem whose difficulty it was not desired to augment by carrying out operations without the protection of a housing, under the low temperatures which prevailed, particularly towards evening and at night. It would have been impossible to carry out observations continuously for a period of 48 hours in the open as was done in the building.

Observations at the summit of Pike's Peak were carried out over a period of a week. The apparatus was then dismantled and transported down to Colorado Springs, where, through the courtesy of President C. C. Mierow of Colorado College, it was set up in a garage with a thin roof and relatively free from surrounding absorbing structures. The building was chosen, to duplicate as far as possible, the conditions in the building used at the summit of Pike's Peak. These observations were carried out for two days. The apparatus was then transported to New Haven where it was set up later in the year and another set of observations was taken in the attic of the Sloane Laboratory, with nothing but a relatively thin roof between the apparatus and the sky.

The following table gives a summary of the locations, and general conditions of the experiments. "Unshielded" refers to the sphere with one inch wall unshielded with lead except for the lead plates at the bottom on which it rested. "Shielded" refers to the condition in which the sphere was surrounded by two inches of lead on all sides.

TABLE I.

Locations and Conditions.

Location and Altitude.	Period of Experiments.	Shielded or Unshielded.	Pressure Range Pounds per sq. inch above atmos. pressure.	Time of Day.
Pike's Peak 14,000 ft.	Sept. 19	Unshielded	512–A	4 P.M.–8 P.M.
	Sept. 20	Shielded	504–A	10 A.M.–5 P.M.
	Sept. 20–21[6]	Shielded	977–A	7.30 P.M.–5 A.M.
Colorado Springs 6,000 ft.	Sept. 24	Unshielded	969–A	9 A.M.–9 P.M.
	Sept. 25	Shielded	984–A	Midnight–3.30 P.M.
New Haven (Sea Level)	Dec. 4–5	Shielded	960–A	9 A.M.–7 P.M. Dec. 4–Dec. 5
	Jan. 8	Unshielded	944–A	12 Noon–3 P.M.
	Jan. 25–27	Shielded	917–A	5 P.M.–10 A.M. Jan. 25–Jan. 27

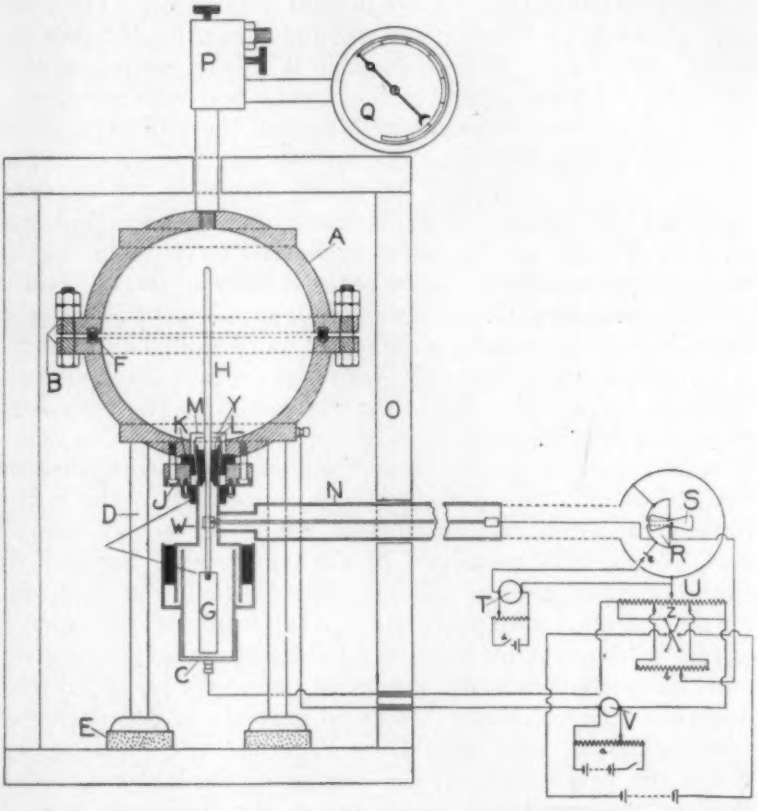
[6] Last observation at atmospheric pressure taken at 10 o'clock on the following day.

THE APPARATUS.

The apparatus used is represented diagramatically in Fig. 1, but with different parts drawn to different scales for convenience. The dead black parts represent hard rubber. *A* is the steel vessel which, with its immediate connections, was surrounded by the lead shield *O*. The vessel *A* was made in two hemispherical parts which could be bolted together. Two recesses were cut in the flanges so that they abutted against each other. The bottom of each recess was lined with lead, and a steel ring fitted into the two recesses as shown in Fig. 1. When the bolts were properly tightened, a very perfect air-tight joint was secured. The parts most difficult to render airtight were those which depended upon the use of hard rubber. The construction shown provided for a satisfactory degree of tightness. *H* is the central insulated rod which is prolonged downwards to the metal cylinder *G*, forming the central member of a condenser, which we shall refer to as the compensating condenser, whose outer member was insulated and provided with an adjustable cap *C*, which served to produce small changes in the capacity. *G* could also be replaced by other cylinders when larger variations in

capacity were desired. The part *W* formed, at its upper end, a guard ring which prevented charge from leaking across the hard rubber from the sphere to the rod *H*. This guard ring was capped by a metal piece *Y*. This, and the metal part *M* which was in metallic connection with the sphere, formed

FIG. 1.



shields which prevented any appreciable part of the rod *H* from being subjected to induction influences of that portion of the hard rubber which separated the guard ring from the sphere. For, across this piece of hard rubber, a high potential was maintained, and, under such conditions, soakage and leakage across a dielectric which is not shielded from an insulated system can, by inductive action, produce very

serious changes in potential of the insulated system in spite of the fact that the guard ring prevents an actual conduction of electricity to the insulated system. For a similar reason, the construction indicated for the compensating condenser was adopted for the purpose of shielding G and its immediate connections from the hard rubber which formed the insulating support of the outer member of that condenser. The lower part of the piece W served as the guard ring for the compensating condenser. The whole piece W was in connection with the tube N which, again, was in connection with the case of the quadrant electrometer which formed the arbitrary zero of potential of the system. The system HG was connected as shown, to one quadrant of an electrometer whose other quadrant was connected to the electrometer case. We shall refer to HG and all the parts connected to it, including the electrometer quadrant, as the *central system* of the apparatus.

The resistance U was a megohm, supplemented by other resistances at its ends for purposes of more refined adjustment. A battery of heavy duty radio batteries giving a total potential difference of about 1,300 volts was joined to the ends of U through a reversing switch in the manner shown. One end of U was connected to the outer member of the compensating condenser, and the other end was connected to the sphere through a potentiometer system which served to insert a variable potential measured by the voltmeter V between the end in question and the sphere. The needle of the electrometer was supplied with potential by a tap from the resistance U, or rather from a shunt b across it, which shunt served to give a more convenient fine adjustment on the needle potential when desired. The voltmeter T formed part of the potentiometer system shown, and served to calibrate the electrometer. When the key d was up, the closing of the key e served to connect the insulated quadrant to the arbitrary zero Z. When d was closed, the closing of e served to apply the potential recorded by T to the insulated quadrant.

The purpose of the compensating condenser in conjunction with the other features above described was to prevent fluctuations of the battery potential from producing spurious deflections of the electrometer through inductive action on the central system. It is obvious that if the coefficient of in-

duction between the two members of the compensating condenser and the coefficient of induction between the sphere and the central system bear to each other the inverse ratio of the potentials applied to C and to A respectively, Z being the arbitrary zero of potential, then, the potential of the central insulated system will be absolutely independent of fluctuations of the potential of the 1,300 volt battery. Strictly speaking, we should include in our discussion the inductive action of the needle on the quadrant, and say that if (1) refers to the central insulated system, (2) to the sphere, (3) to the outer member of the compensating condenser, and (4) to the electrometer needle; and, if V_2, V_3, V_4 refer to the potentials of the last three with the potential of Z taken as an arbitrary zero, then the condition for freedom from effects due to battery fluctuations is

$$q_{21}V_2 + q_{31}V_3 + q_{41}V_4 = 0,$$

when the q's are the corresponding coefficients of induction. Adjustment to secure this condition was made by adjusting the position of the point Z along the resistance U, final adjustments being made by means of the smaller resistance at the ends of U, and by adjustment of the distance of the cap C from the part G. The compensations were made until, on insulating the central system and reversing the reversing switch, no deflection of the electrometer was obtained. In view of the variation of the specific inductive capacity of air with pressure, the compensation had to be remade for each pressure used. This was most effectively done by means of the adjusting resistance at the ends of U, and still more finely by means of the adjustable cap C.

In providing for a compensation of this kind, it is highly desirable to insure that the resistance ratios contained in the resistance U shall remain constant. For this reason, it is usually undesirable actually to *earth* the point Z; for then, if there is leakage from the ends of the battery to earth, or from the sphere or reversing key to earth, the effective resistances between Z and the ends of U will be altered to extents which are variable and dependent upon the vagaries of the leakage paths. In order to avoid complications of this kind, the legs which supported the sphere were supported on sulphur pieces

E, the electrometer legs were supported on three sulphur pillars, and the potentiometer systems associated with T and with V were mounted on separate sulphur insulated boards, as was also the resistance U with its immediate connections.

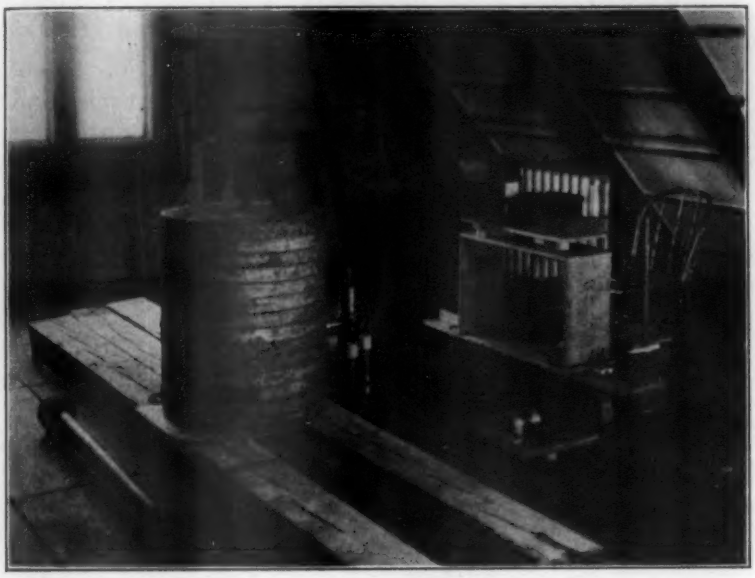
FIG. 2.

The actual ionization currents in the sphere were measured by balancing the electrometer deflections by gradually altering V by means of the variable resistance a. This resistance was, in fact, a Pye rotating wire potentiometer supplemented by a sulphur insulated crank handle.

Figs. 2 and 3 show pictures of the actual apparatus as set up at New Haven. In Fig. 2, some of the lead rings are removed, showing the steel sphere within. The attachment above the top lead disc contained a needle valve, and the necessary unions for attachment to the gauge and to the inlet supply from the high pressure cylinder. In filling the sphere, the air from the high pressure cylinder was allowed to pass through a long iron cylinder shown to the left of Figs. 2 and 3. The greater part of the length of this cylinder was occupied by a metal boat filled with phosphorous pentoxide to

dry the air. The boat was broken up into sections by partitions which nearly filled the cylinder, so as to insure thorough mixing of the air in passing through. The exit end of the cylinder was packed with cotton to take out dust and phos-

FIG. 3.



phorous pentoxide powder. The air was allowed to pass through this cylinder very slowly so as to permit of thorough drying; that the drying was very thorough was shown by the fact that the phosphorous pentoxide at the exit end of the tube was perfectly dry after use. The sphere was washed out with dry air before being filled for the experiments; and, of course, was disconnected from the gas supply during the measurements.

The significance of the various parts of the apparatus in Fig. 2 will be self evident. Just below the iron sphere will be seen the compensator, partly visible above the lead rings. The electrometer, which was of the type designed by the writer [7] is seen standing on its three sulphur supports. Im-

[7] "An Attempt to Detect a Corpuscular Radiation of Cosmic Origin," JOURNAL FRANKLIN INSTITUTE, Vol. 203, Jan. 1927.

mediately to the right of it is the megohm and its associated resistances. Further in the foreground is the calibrating system on its sulphur insulated board; and, mounted on an inverted U shaped wooden structure is the sulphur insulated platform which carried the rotating potentiometer which, however, is not shown in the picture.

MEASUREMENT OF IONIZATION CURRENTS.

The procedure in taking a measurement was first to note the zero θ of the electrometer. The key e was then raised and the electrometer spot started to move. The potentiometer handle was cranked so as to bring the reading to θ; and, after it had been held there for half a minute, the voltmeter reading V_1 was taken, and at the same instant, a stop watch was started. The cranking was continued so as to keep the reading at θ and so avoid electrometer leakage. In case the spot should go inadvertently to the right of θ for a certain time during the operation, it was afterwards brought to an apparently equal distance to the left for the same time during the same measurement. Then, when a sufficiently large reading had been obtained on V, great care was taken to hold the electrometer reading exactly at θ, for a period of about half a minute. The voltmeter reading V_2 was then taken and at the same instant, the watch was stopped. The electrometer key was then closed.

If T is the time interval between the two readings V_1 and V_2, Q the rate of production of ions per cc. in the vessel, W the volume of the vessel, and e the electronic charge, we have, if V_2 and V_1 are in volts,

$$QWe = q_{21}\frac{(V_2 - V_1)}{300T},$$

the positive values of e and of $V_2 - V_1$ being of course inserted. In each case the reading was repeated about three times in accordance with the general plan outlined below.

GENERAL EXPERIMENTAL PROCEDURE.

While the apparatus had been thoroughly tested for gas leaks before shipment to Pike's Peak, leakage troubles were encountered at one of the hard rubber joints when it was set up on the mountain; and, the conditions were not such as to

make it easy to remedy them as effectively as might have been desired. For this reason, precautions were taken to minimize their effects. The sphere was blown up to the maximum pressure desired; and the electrical compensation was made. A period of about twenty minutes was allowed to elapse to enable the gas to come to a steady temperature. The pressure gauge Q was then read and the time T_1 was noted. Three, and sometimes more successive observations of the ionization were then taken, and the gauge and time T_2 were again read. From the gauge readings at T_1 and T_2 the fall of pressure per second was obtained. Let us call this R. The sum of the times τ for the three (or more) observations was obtained by addition of individual times for each. Then, after each separate observation a period of 75 seconds was required to prepare for the next. This time for preparation became very constant after the routine had become established. Thus, if there were n observations, the total time consumed from the start of the first to the completion of the last was $(\tau + 75(n - 1))$ seconds. If P_0 is the initial gauge reading, the average gauge reading during the observations was then $P_0 - \{\tau + 75(n - 1)\}\frac{R}{2}$. The values so obtained were corrected from the calibration curve of the gauge, which curve was determined after the experiments were completed. In this way, the values $P_1 = P_0 - \{\tau + 75(n - 1)\}\frac{R}{2} + C$ were determined, where C is the calibration correction. During the final observations at New Haven, the apparatus was made sufficiently tight so that the correction for leakage was unnecessary.

In order to obtain a quantity representative of the amount of gas in the sphere, the quantities P_1 were corrected for the atmospheric pressure as read from a barometer. In this way the quantity which we shall call P_2 was obtained. Finally, the values of P_2 were reduced to a common temperature [8] of 45° F. (7.2° C.), and the values so obtained will be called the corrected pressures P.

[8] The temperatures at Pike's Peak during the experiments were in the neighborhood of 45° F.

It was of course very important to insure that the potential was high enough to secure saturation, and a special test of this matter was made. The difficulty of securing saturation increases, with the strength of the ionization. The saturation test was made at New Haven; but, in order to render it as stringent as possible, the ionization during the test was increased to about twice the value obtained for the lead shielded experiments for the highest pressure used at the top of Pike's Peak. The increase of ionization was brought about by means of the gamma rays from 1.3 milligrams of radium placed at a distance of about three feet from the sphere. Using this enhanced ionization, the procedure was to measure, at the highest pressure used, the ionization with a potential difference of 1,250 volts between the sphere and the central system, this being about the voltage used in the actual measurements. The experiment was then repeated with a potential difference of 2,500 volts. The results of this test are given in Table II.

TABLE II.

Saturation Tests.

D = Distance of *Ra* from sphere; J = Potential applied to sphere; P = Pressure in pounds per square inch, corrected for atmospheric pressure, gauge calibration, and corrected also to a temperature of 7.2° C.; I_1 = Ions per cc. per second; I = mean of values of I_1 for readings indicated.

D.	J.	P.	Temp.	I_1.	I.
120 cms.	1,250	935	19.3° C.	126	
	"	"	"	126	126
	"	930	18.9° C.	126	126
120 cms.	2,500	930	20° C.	128	
	"	"	"	127	127
	"	"	"	127	
	"	"	"	127	
	"	"	20.8° C.	130	130
90 cms.	1,250	915	22° C.	223, 226	225
		"	"	228	228
90 cms.	2,500	917	21° C.	233	
		"	"	235	233
				233	

Taking the data above the double line in Table II, we see that the value of I for 1,250 volts agrees very well with that

for 2,500 volts, and a similar remark applies to the other set of data below the double line. That the small variation of pressure during either of these sets is insignificant in its effect on the conclusion is evident from the observations themselves which, for a change of 5 lbs. per sq. inch in the case of the observations above the double line, show no appreciable change in the measured ionization.

In view of the fact that the test applies to conditions so much more stringent than those encountered in the main experiments, it is concluded that saturation was amply provided for with the voltages used.

DETERMINATION OF COEFFICIENT OF INDUCTION q_{21} BETWEEN THE SPHERE AND THE CENTRAL SYSTEM.

The plan was to connect to the central system the inner member of a subsidiary condenser whose coefficient of induction q'_{21}, between its inner and outer members, was known. The electrometer key having been raised, with the sphere and the outer member of the subsidiary condenser connected to the electrometer case, a potential V was applied to the sphere and a potential U to the outer member of the subsidiary condenser. U was then adjusted so as to bring the electrometer reading back to the value which it had when the key was raised. Under these conditions

$$q_{21}V = q'_{21}U,$$

so that q_{21} is determined in terms of known quantities.

In order to avoid errors caused by ionization current leakage in the sphere during the adjustment of U, it is desirable to secure the final adjustment of U in a manner slightly different from that above described. The potentials V and U which have been approximately determined, as above, are applied while the electrometer quadrant is connected to the case. The quadrant is then disconnected from the case and immediately the sphere and outer member of the subsidiary condenser are connected to the electrometer case. If there is no movement of the spot as a result of reducing V and U to zero, we know that the adjustment has been properly made; the results are not complicated by the flow of ionization current during the time one is observing the electrometer spot

in order to ascertain whether or not it is steady. It was by this process that the final adjustments of U were made.

The coefficient of induction, q'_{21}, for the subsidiary condenser was capable of being varied, so a set of ten observations with different values of V and different values of q'_{21} were made in order to determine q_{21}. The average of the variation of individual observations from the mean value was of the order of one per cent. of the mean.

Owing to a change in the central system after the observations at Pike's Peak, the value of q_{21} for that situation was different from the corresponding value for Colorado Springs and New Haven. The values are as follows:

Pike's Peak..... $q_{21} = 4.24$ cms.
Colorado Springs and New Haven... $q_{21} = 4.18$ cms.

DEFINITION OF SYMBOLS.

P_0 = Actual initial gauge reading in pounds per square inch.
ΔP = Fall of pressure in time ΔT.
$R = \frac{\Delta P}{\Delta T}$ = Fall of pressure per second.
τ = Sum of times occupied in actual measurements of ionization currents in any one set.
n = Number of observations in any one set.
$\overline{P} = P_0 - [\tau + 75(n-1)]\frac{R}{2}$ = Average gauge reading during one set.
$P_1 = P_0 - [\tau + 75(n-1)]\frac{R}{2} + C$ Average excess pressure over atmospheric pressure corrected for gauge calibrations by the quantity C.
p = Atmospheric pressure in pounds per square inch.
P_2 = Average pressure in sphere during set = $P_1 + p$.
t = Temperature during set.
P = Average pressure in sphere during set, reduced to a temperature of 45° Fahrenheit (7.2° C.). $P = P_2[1 + 3.6 \times 10^{-3}(t - 7.2°)]$.
$V_2 - V_1$ = Change in reading of the voltmeter V during interval T.
T = Time interval in seconds corresponding to the change $V_2 - V_1$ in the voltmeter V.

q_{21} = Coefficient of induction between sphere and central system.

e = Electronic charge (positive value) = 4.77×10^{-10} e.s.u.

W = Volume of the sphere.

Q = Number of ions produced per cc. per second.

$$Q = \frac{q_{21}}{300We} \frac{V_2 - V_1}{T}.$$

ϵ = The average percentage deviation (regardless of sign) of the observations from the mean for the corresponding pressure P_0.

RESULTS.

Table III gives in detail an illustration of one complete run in the main experiment, the various magnitudes involved in the correction being shown.

It must be observed that these data apply to a case where the gas leakage was most pronounced. The accuracy of the observations does not, of course, warrant the expression to four figures, but, for convenience of computation and uniformity, four figures have been given except in the final column.

Tables IV–XI give the condensed observations. The values of ϵ are not recorded but their magnitudes will be evident from a glance at the third and fourth columns.

The data corresponding to the cases in which the apparatus was unshielded, have no fundamental importance, but are given as matters of general interest.

The curves corresponding to the foregoing data are shown in Fig. 4. Their significances will be clear from the legend attached. Two sets of observations were obtained for the shielded condition at Pike's Peak, but as one set extended to a pressure of only 504 lbs. per sq. inch, while the other extended to 982 lbs. per sq. inch, only the latter data are plotted. The former data, however, fall on the curve of the latter within the accuracy of the experiment, and within the limits of uncertainty imposed in drawing the curves. The two sets of data for the shielded case at New Haven fell so closely on one curve that it was impossible to distinguish between them.

For convenience of discussion of certain aspects of the results, it is convenient to construct empirical algebraic

TABLE III.

Typical Table of Actual Observations.

Pike's Peak, apparatus shielded with lead.

P_0.	ΔP.	ΔT.	R.	τ.	$\overline{P}$.	P.	$V_2 - V_1$.	T.	$\frac{V_2 - V_1}{T}$.	Average $\frac{V_2 - V_1}{T}$.	ϵ.	t.	p.
1,010	55.0	1,560	0.035	832	992	982	11.70	182.5	0.06440	0.06326	1.7	7.7° C.	9.0
							14.30	227.0	0.06300		0.4		
							13.45	216.5	0.06212		1.7		
							13.15	206.0	0.06383		0.9		
900	45.0	1,560	0.029	833	885	878	12.80	203.0	0.06305	0.06224	1.3	7.7° C.	9.0
							12.85	211.0	0.06090		2.1		
							13.60	215.5	0.06310		1.4		
							12.60	203.5	0.06192		0.5		
805	33.0	1,200	0.027	636	795	800	12.15	200.0	0.06075	0.05942	2.3	7.7° C.	9.0
							11.40	193.0	0.05907		0.6		
							14.20	243.0	0.05843		1.7		
700	28.0	1,440	0.019	598	693	700	11.00	195.0	0.05641	0.05673	0.6	7.7° C.	9.0
							11.30	199.5	0.05664		0.2		
							11.60	203.0	0.05714		0.7		
602	33.0	1,980	0.017	954	592	608	10.35	195.5	0.05294	0.05413	2.2	7.7° C.	9.0
							11.80	213.5	0.05527		2.1		
							11.20	200.5	0.05586		3.2		
							7.40	140.5	0.05267		2.7		
							11.00	204.0	0.05392		0.4		

TABLE III.—*Continued.*

P_0.	ΔP.	ΔT.	R.	r.	$\overline{P}$.	P.	$V_2 - V_1$.	T.	$\frac{V_2 - V_1}{T}$.	Average $\frac{V_2 - V_1}{T}$.	ϵ.	t.	p.
495	30.0	1,680	0.018	981	482	503	15.70 16.95 16.60	313.5 336.5 331.0	0.05007 0.05037 0.05015	0.05020	0.3 0.3 0.1	7.7° C.	9.0
400	13.0	1,620	0.0080	776	396	423	12.1 11.35 12.50	261.0 249.0 265.5	0.04636 0.04558 0.04708	0.04634	0.0 1.7 1.6	7.7° C.	9.0
300	13.0	2,160	0.0060	1,239	296	333	12.10 12.90 12.50 12.55	304.5 314.0 310.5 310.0	0.03973 0.04108 0.04025 0.04048	0.04038	1.6 1.8 0.3 0.2	7.6° C.	9.0
200	7.5	2,580	0.0029	1,564	197	240	11.30 10.75 10.75 10.30 9.60	313.5 311.5 312.0 314.5 312.0	0.03604 0.03451 0.03445 0.03275 0.03397	0.03434	5.0 0.5 0.3 4.7 1.1	7.4° C.	9.0
100	5.0	2,160	0.0023	1,324	98.0	148	12.10 11.90 12.20	477.0 471.0 475.5	0.02537 0.02526 0.02566	0.02543	0.2 0.7 0.9	6.6° C.	9.0
50	2.5	3,240	0.0008	1,824	49.2	104	11.50 11.95 11.80	602.0 601.0 621.0	0.01910 0.01988 0.01900	0.01933	1.2 2.9 1.7	6.6° C.	9.0
(Atmos. Press.) 9						9	7.60 7.35	178.4 182.9	0.00426 0.00402	0.00414	3.0 3.0	6.6° C.	9.0

TABLE IV.

Date: September 19, 1926.

(Apparatus was Unshielded.)

Location and Elevation.	P.	$\frac{V_2 - V_1}{T}$.	Average $\frac{V_2 - V_1}{T}$.	Ions per cc./sec.
Pike's Peak, 14,000 feet	511	0.1142 0.1129 0.1148	0.1141	228
	463	0.1126 0.1115 0.1110	0.1117	224
	399	0.1020 0.1009 0.1021	0.1017	204
	332	0.09355 0.09222 0.09220	0.09266	186
	238	0.07644 0.07654 0.07768	0.07688	154
	147	0.05831 0.05788 0.05516 0.05469	0.05651	113
	(Atmospheric Pressure) 9	0.006947 0.006745	0.006846	13.7

TABLE V.

Date: September 20, 1926.

(Apparatus was Shielded with Lead.)

Location and Elevation.	P.	$\frac{V_2 - V_1}{T}$.	Average $\frac{V_2 - V_1}{T}$.	Ions per cc./sec.
Pike's Peak, 14,000 feet	982	0.06420 0.06300 0.06212 0.06383	0.06326	127
	878	0.06305 0.06090 0.06310 0.06192	0.06224	125
	800	0.06075 0.05907 0.05843	0.05941	119
	700	0.05641 0.05664 0.05714	0.05673	114
	608	0.05294 0.05527 0.05586 0.05267 0.05392	0.05413	108
	503	0.05007 0.05037 0.05015	0.05020	101
	423	0.04636 0.04558 0.04708	0.04634	92.8
	333	0.03973 0.04108 0.04025 0.04048	0.04038	80.9
	240	0.03604 0.03451 0.03445 0.03275 0.03397	0.03434	68.8
	148	0.02537 0.02526 0.02566	0.02543	50.9
	104	0.01910 0.01988 0.01900	0.01933	38.7
	(Atmospheric Pressure) 9	0.00426 0.00402	0.00414	8.29

TABLE VI.

Date: September 20, 1926.

(Apparatus was Shielded with Lead.)

Location and Altitude.	P.	$\frac{V_2 - V_1}{T}$.	Average $\frac{V_2 - V_1}{T}$.	Ions per cc./sec.
Pike's Peak, 14,000 feet	504	0.04976 0.05176 0.05096	0.05083	102.0
	447	0.04661 0.04794 0.04696	0.04717	94.4
	378	0.04247 0.04500 0.04437 0.04474	0.04414	88.4
	286	0.03847 0.03667 0.03768 0.03643	0.03731	74.7
	222	0.03200 0.03170 0.03264	0.03211	64.3
	149	0.02547 0.02523 0.02476	0.02515	50.4
	104	0.01921 0.01870 0.01932	0.01908	38.2
	(Atmospheric Pressure) 9	0.00426 0.00407	0.00416	8.34

TABLE VII.
Date: September 24, 1926.
(Apparatus Unshielded.)

Location and Elevation.	P.	$\frac{V_2 - V_1}{T}$.	Average $\frac{V_2 - V_1}{T}$.	Ions per cc./sec.
Colorado Springs, 6,000 feet	946	0.06318 0.06178 0.06274	0.06256	123
	849	0.06070 0.06370 0.06029 0.05934 0.06155 0.06054 0.06062	0.06096	119
	781	0.05762 0.06059 0.06083 0.06121	0.06051	119
	686	0.05650 0.05564 0.05563	0.05591	110
	600	0.05354 0.05199 0.05292 0.05349	0.05298	105
	506	0.04869 0.04980 0.04836 0.04990	0.04919	97.1
	418	0.04553 0.04472 0.04410	0.04478	88.4
	331	0.04091 0.03969 0.03905 0.03898	0.03966	78.3
	236	0.03198 0.03139 0.03225	0.03183	62.8
	147	0.02476 0.02358 0.02340 0.02377	0.02388	47.1
	106	0.01813 0.01780 0.01725	0.01773	35.0
	(Atmospheric Pressure) 12	0.004263 0.004350	0.00431	8.50

TABLE VIII.

Date: September 24, 1926.

(Apparatus Shielded with Lead.)

Location and Elevation.	P.	$\frac{V_2 - V_1}{T}$.	Average $\frac{V_2 - V_1}{T}$.	Ions per cc./sec.
Colorado Springs, 6,000 feet	982	0.02859 0.02866 0.02952	0.02892	57.1
	910	0.02778 0.02882 0.02775 0.02898	0.02833	55.9
	835	0.02698 0.02816 0.02720 0.02751	0.02746	54.1
	745	0.02689 0.02594 0.02655 0.02671	0.02652	52.4
	653	0.02540 0.02555 0.02468 0.02484	0.02512	49.6
	564	0.02401 0.02423 0.02359	0.02394	47.3
	469	0.02186 0.02180 0.02170	0.02179	43.0
	379	0.01917 0.01920 0.01954	0.01930	38.1
	289	0.01715 0.01717 0.01673	0.01702	33.6
	197	0.01348 0.01317 0.01312	0.01326	26.2
	148	0.01122 0.01090 0.01113	0.01108	21.9
	107.5	0.008571 0.008833	0.008702	17.2
	(Atmospheric Pressure) 12	0.003011 0.002977	0.002994	5.91

TABLE IX.

Date: January 8, 9, 1927.

(Apparatus Unshielded.)

Location and Elevation.	P.	$\frac{V_2 - V_1}{T}$.	Average $\frac{V_2 - V_1}{T}$.	Ions per cc./sec.
New Haven, Sea Level	944	0.03625 0.03518 0.03567 0.03640	0.03587	70.8
	862	0.03516 0.03485 0.03582	0.03528	69.6
	783	0.03341 0.03477 0.03310 0.03441	0.03392	67.0
	683	0.03189 0.03239 0.03318 0.03203	0.03237	63.9
	596	0.03144 0.03198 0.03093 0.03104	0.03134	61.9
	503	0.02824 0.02800 0.02786	0.02803	55.3
	415	0.02537 0.02582 0.02605	0.02575	50.8
	329	0.02317 0.02225 0.02239	0.02260	44.6
	240	0.01884 0.01863 0.01882	0.01876	37.0
	195	0.01660 0.01650 0.01672	0.01661	32.8
	151	0.01385 0.01407 0.01385	0.01392	27.5
	108	0.01065 0.01068 0.01070	0.01069	20.9
	(Atmospheric Pressure) 15	0.003612 0.003706 0.003683	0.003667	7.24

TABLE X.

(Apparatus Shielded with Lead.)

Location and Elevation.	P.	$\frac{V_2 - V_1}{T}$.	Average $\frac{V_2 - V_1}{T}$.	Ions per cc./sec.
New Haven, Sea Level	960	0.01976 0.01987 0.01932 0.02031 0.02034	0.01992	39.3
	861	0.01937 0.01961 0.01916 0.01936	0.01938	38.3
	773	0.01817 0.01724 0.01808 0.01854	0.01801	35.6
	691	0.01715 0.01685 0.01760 0.01650 0.01740	0.01712	33.8
	598	0.01638 0.01580 0.01659	0.01626	32.1
	505	0.01512 0.01538 0.01470	0.01507	29.8
	417	0.01331 0.01366 0.01340 0.01348	0.01346	26.6
	334	0.01201 0.01190 0.01244	0.01212	23.9
	242	0.00989 0.00967 0.01014	0.00990	19.5
	199	0.00870 0.00878 0.00883	0.00877	17.3
	152	0.00749 0.00691 0.00752 0.00738	0.00733	14.5
	109	0.00631 0.00587 0.00576 0.00594	0.00599	11.8
	(Atmospheric Pressure) 15	0.002525 0.002627 0.002747 0.002627	0.00263	5.19

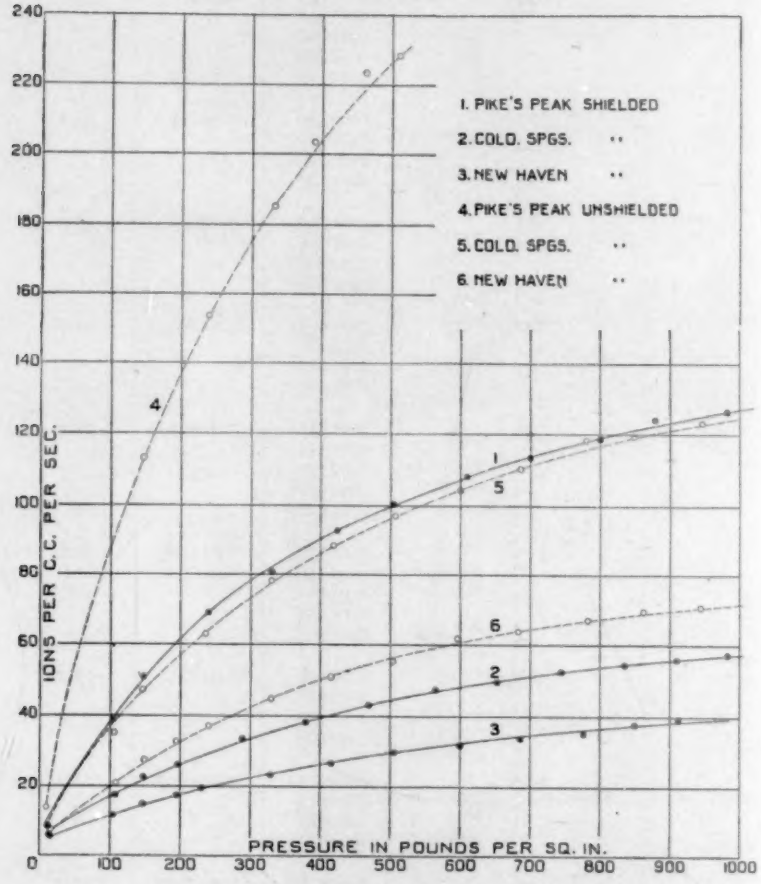
TABLE XI.

(Apparatus Shielded with Lead.)

Location and Elevation.	P.	$\frac{V_2 - V_1}{T}$.	Average $\frac{V_2 - V_1}{T}$.	Ions per cc./sec.
New Haven, Sea Level	912	0.01964 0.01992 0.01982 0.02021 0.02004	0.01992	39.3
	849	0.01963 0.01946 0.01890 0.01858 0.01920	0.01916	37.8
	777	0.01739 0.01792 0.01799 0.01785	0.01779	35.1
	687	0.01730 0.01739 0.01696 0.01696	0.01715	33.9
	600	0.01631 0.01587 0.01615 0.01593	0.01606	31.7
	505	0.01530 0.01551 0.01479 0.01518 0.01532	0.01522	30.0
	415	0.01353 0.01430 0.01291 0.01353 0.01363	0.01358	26.8
	328	0.01170 0.01187 0.01156 0.01189	0.01176	23.2
	237	0.009711 0.01013 0.00987 0.00988	0.009897	19.5
	195	0.008767 0.008788 0.008647	0.008734	17.2
	147	0.007491 0.007554 0.007436	0.007493	14.8
	104	0.00575 0.00588 0.00589	0.005840	11.5
	(Atmospheric Pressure) 15	0.002676 0.002636 0.002582	0.002631	5.19

expressions to represent the curves. The formulæ given in Table XII represent the data to an accuracy of better than one per cent. of the highest ionization; and, if these empirical

FIG. 4.



formulæ were plotted, they would represent the data within the limits of possibility of drawing the curve to fit the experimental results. In addition to giving I as a function of P, the derivative dI/dP is also given, since this quantity figures in the discussion of the result.

TABLE XII.

Empirical Formulæ.

Location.	Condition.	I.	$\frac{dI}{dP}$.
Pike's Peak	Shielded	$138.2 - 132.3e^{-0.00347P} + 8.33e^{-0.003P} \times \sin\frac{2\pi(P-10)}{1000}$	$+ 0.3268e^{-0.00347P} - 0.025e^{-0.003P} \times \sin\frac{2\pi(P-10)}{1000} + 0.0523e^{-0.003P} \times \cos\frac{2\pi(P-10)}{1000}$
Colorado Spgs.	Shielded	$64 - 58.3e^{-0.0021P}$	$0.1259e^{-0.00216P}$
New Haven	Shielded	$45 - 41.06e^{-0.002P}$	$0.0811e^{-0.002P}$

Table XIII gives the values of I and dI/dP for each 100 lbs. per sq. in. of pressure over the range used, the values being calculated from the formulæ given in Table XII.

TABLE XIII.

SUMMARY OF EXPERIMENTAL DATA.

P.	Pike's Peak, Shielded.		Colorado Springs (Shielded).		New Haven (Shielded).	
	I.	$\frac{dI}{dp}$.	I.	$\frac{dI}{dp}$.	I.	$\frac{dI}{dp}$.
100	38.5	0.278	17.4	0.101	11.4	0.0664
200	61.9	0.197	26.1	0.0815	17.5	0.0543
300	78.4	0.141	33.6	0.0658	22.5	0.0445
400	90.6	0.120	40.1	0.0530	26.6	0.0364
500	100.0	0.0830	44.2	0.0428	29.9	0.0298
600	107.0	0.0695	48.1	0.0344	32.6	0.0244
700	114.0	0.0584	51.1	0.0278	34.9	0.0200
800	119.0	0.0484	53.7	0.0223	36.7	0.0164
900	123.0	0.0394	55.6	0.0181	38.2	0.0134
1,000	127.0	0.0303	57.3	0.0145	39.5	0.0109

The data contained in Table XIII may be said to represent the experimental results free from all theories of the nature and origin of the ionization. For the interpretation of them, and for the purpose of making allowance for absorption of radiation by the walls of the room, the lead shields, etc., we must enter upon more speculative grounds.

CORRECTION FOR ABSORPTION BY SHIELDS, AND INTERPRETATION OF RESULTS IN RELATION TO COSMIC RADIATION.

Our first desire would naturally be to correct the results for absorption by shields, etc., and then to discuss the significance of the data at different altitudes in relation to absorption of cosmic radiation by the atmosphere. Unfortunately, however, it is impossible to dissect the problem into two parts in this manner; for, the absorption by shields cannot properly be considered without an assumption as to the composition of the radiation.

The shields comprised in the first place the iron sphere itself. Then, there was the 2″ casing of lead, together with the lead disc at the top. Finally, there was the building in which the apparatus was housed. The dimensions and materials here concerned were of course different for Pike's Peak and Colorado Springs. The experiment at New Haven was performed in an attic where the roof was sufficiently thin to warrant neglecting it entirely. Any effect of absorption by a roof would be less important in the New Haven observations than it would have been in the others on account of the fact that the importance of the harder radiation is enhanced at New Haven in relation to its value at the other stations. The procedure adopted in allowing for absorption was as follows:

(1) It was assumed that the cosmic radiation at any point in the atmosphere is made up of a number of parts, of different frequencies, the number dN of quanta of any one frequency ν, coming per unit area to an element of surface ds per second from directions contained within the solid angle $d\omega$, to whose axis ds is perpendicular, is

$$dN_\nu = C_\nu e^{-(H\mu_\nu/\cos\theta)} d\omega \qquad (1)$$

where θ is the angle made by the axis of $d\omega$ with the vertical, H is the height of the homogeneous atmosphere in meters of water above the point of observation, μ_ν is the coefficient of absorption for the frequency ν expressed in terms of meters of water C_ν is a constant determined by the condition that

$$N_\nu = 2\pi C_\nu \int_0^{(\pi/2)} \sin\theta d\theta = 2\pi C_\nu, \qquad (2)$$

where N_ν is the number of quanta of frequency ν passing through one square centimeter with vertical normal, and situated at a point so high that absorption above that point is negligible. From (1) and (2) we have

$$dN_\nu = \frac{N_\nu}{2\pi} e^{-(H\mu_\nu/\cos\theta)} \sin\theta d\theta d\varphi. \qquad (3)$$

The number of these quanta absorbed per unit volume at atmospheric pressure is

$$dN_{\nu o} = \frac{D_a}{100}\mu_\nu dN_\nu = \frac{D_a N_\nu \mu_\nu}{2\pi \times 100} e^{-(H\mu_\nu/\cos\theta)} \sin\theta d\theta d\varphi, \qquad (4)$$

where D_a is the density of air at atmospheric pressure, and at the temperature of the experiment.

(2) The corresponding value of $dN_{\nu o}$ for a point at atmospheric pressure inside the sphere was calculated as a function of the coefficients of absorption and thicknesses of the various intervening materials. The coefficients of absorption were of course functions of ν and they were assumed proportional to the densities of the materials. In a manner to be more specifically outlined below, the procedure was to calculate the ratio of the value obtained by integrating (4) from $\theta = 0$ to $\theta = \pi/2$, and from $\varphi = 0$ to $\varphi = 2\pi$, to the value obtained for the corresponding quantity for a point inside the vessel. This ratio, which is a function of ν and which we write $R_{H\nu}$ represents the factor by which the absorption of quanta of frequency ν per second in the vessel, at one atmosphere, must be multiplied in order to obtain the corresponding quantity for a point outside the vessel. $R_{H\nu}$ was a function of the station, since the surrounding walls were of course different for Pike's Peak, Colorado Springs and New Haven.

(3) To obtain the average ionization corresponding to a quantum of frequency ν, we must multiply the corresponding number of quanta absorbed by $h\nu/Ue$, where U is the ionization potential, and e the electronic charge. Thus, using (4), the total contribution to the ionization at one atmosphere inside the vessel by the quantity of frequency ν, is

$$I_{H\nu} = \frac{D_a N_\nu \mu_\nu h\nu}{2\pi e U R_{H\nu} \times 100} \int_0^{2\pi} d\varphi \int_0^{(\pi/2)} e^{-(H\mu_\nu/\cos\theta)} \sin\theta d\theta;$$

making the transformation $\cos\theta = 1/x$, we have

$$I_{H\nu} = \frac{D_a N_\nu \mu_\nu h\nu}{eUR_{H\nu} \times 100} \int_1^\infty \frac{e^{-H\mu_\nu x}}{x^2} dx.$$

Writing

$$\frac{D_a N_\nu \mu_\nu h\nu}{100 Ue} = G_\nu$$

we have

$$I_{H\nu} = \frac{G_\nu}{R_{H\nu}} \int_1^\infty \frac{e^{-H\mu_\nu x}}{x^2} dx.$$

Thus, if there be several frequencies involved, the total number of ions produced per cc. at one atmosphere, at a point inside the vessel, is

$$I_H = \sum_\nu \frac{G_\nu}{R_{H\nu}} \int_1^\infty \frac{e^{-H\mu_\nu x}}{x^2} dx$$

or, writing

$$J_{H\nu} = \int_1^\infty \frac{e^{-H\mu_\nu x}}{x^2} dx \tag{5}$$

we have

$$I_H = \sum \frac{G_\nu}{R_{H\nu}} J_{H\nu}. \tag{6}$$

The general procedure was then to endeavor to assign ν's and G_ν's so that the three values calculated for I_H for the three different altitudes agreed with the experimental data. It was further desirable to ascertain the extent to which the assignment was unique. In order to utilize the experimental data in the most efficient manner for the foregoing ends, it was desirable to combine the observations in a certain manner which will be outlined in detail below.

Calculation of $R_{H\nu}$. If $f(\theta, \varphi)$ represents the factor by which $dN_{\nu 0}$ of equation (4) must be multiplied in order to obtain the corresponding quantity for a point inside the vessel, the quantity $R_{H\nu}$ may be represented analytically as

$$R_{H\nu} = \frac{\int_0^{(\pi/2)} \int_0^{2\pi} e^{-(H\mu_\nu/\cos\theta)} \sin\theta f(\theta, \varphi) d\theta d\varphi}{\int_0^{(\pi/2)} \int_0^{2\pi} e^{-(H\mu_\nu/\cos\theta)} \sin\theta d\theta d\varphi}$$

or

$$R_{H\nu} = \frac{\int_1^\infty e^{-(H\mu_\nu/\cos\theta)} \sin\theta d\theta \int_0^{2\pi} \frac{f(\theta, \varphi)}{2\pi} d\varphi}{\int_1^\infty e^{-(H\mu_\nu/\cos\theta)} \sin\theta d\theta}. \quad (7)$$

If $\mu_{\nu_1}, \mu_{\nu_2}, \cdots$ etc., referred to the coefficients of absorption of the various materials, and if $l_1, l_2, \cdots$ etc., refer to the corresponding length of absorbing path for the angles θ and φ,

$$\begin{aligned} f(\theta, \varphi) &= e^{-\mu_{\nu_1} l_1} e^{-\mu_{\nu_2} l_2} \cdots \\ &= e^{-(\mu_{\nu_1} l_1 + \mu_{\nu_2} l_2 + \cdots)}. \end{aligned}$$

Or, if μ_ν refers to the coefficient of absorption per meter of water, and if D refers to the density of the substance,

$$f(\theta, \varphi) = e^{-\mu_\nu (D_1 l_1 + D_2 l_2 + \cdots)}. \quad (8)$$

Considering the range $\theta = 0$ to $\theta = \pi/2$ divided into zones of $5°$ angles, it was possible to assign a μ_ν, and to calculate for any zone the average value of $f(\theta, \varphi)$ as given by (8). In other words, it was possible to calculate for each zone in θ the quantity

$$\int_0^{2\pi} \frac{f(\theta, \varphi)}{2\pi} d\varphi,$$

occurring in Equation (7). By plotting the integrand of the numerator of (7) as a function of θ, it was then possible to evaluate this numerator graphically for the assigned μ_ν. The denominator may be thrown into the form

$$\int_1^\infty \frac{e^{-H\mu_\nu x}}{x^2} dx,$$

where $\cos\theta = 1/x$; and the values of this integral for various values of $H\mu_\nu$ have been tabulated.

The curve of Fig. 5 gives the values of $R_{H\nu}$ as a function of μ_ν for the different values of H involved.

Utilization of the Experimental Data. It must be remarked that, quite apart from the question of the absorption by the walls of the vessel, the significance of the ionization in the closed vessel in relation to the ionization in the open is not one which is immediately clear. By taking the increase in ioniza-

tion per pound increase in pressure at a pressure well above atmospheric pressure we avoid the spurious effect caused by short range radiation from the walls of the vessel. However, the very fact that the ionization-pressure curve is not linear shows that the increase of ionization per lb. is not numerically equal to the quantity which occupies the position of I_H in equation (6). However, it is probably correct to assume that the various terms of the summation in (6) contribute to the measured values of the increase of ionization per cc. per lb. increase in the vessel quantities proportional to the terms in question, the factor of proportionality being a function of the pressure in the vessel and possibly of ν, so that

$$\frac{dI_{HP}}{dP} = \sum_{\nu} \frac{G_\nu B_{\nu P}}{R_{H\nu}} J_{H\nu}, \tag{9}$$

where $B_{\nu P}$ is the function in question, and dI_{HP}/dP is the measured increase in ions per cc. per second per lb. per sq. inch increase of pressure at the pressure P and at the altitude determined by H.

In order to make use of a range of observation from our ionization-pressure curve, we may take the average values of each side of (9) over range P_n to P_m.

$$\frac{1}{P_m - P_n} \int_{P_n}^{P_m} \frac{dI_{HP}}{dP} dP = \sum_{\nu} \frac{G_\nu \bar{B}_{\nu P_{mn}}}{R_{H\nu}} J_{H\nu}, \tag{10}$$

where $\bar{B}_{\nu P_{mn}}$ is the average value of $B_{\nu P}$ over the range P_n to P_m. The left hand side of (10) is integrable immediately, and leads to

$$\frac{I_{HP_m} - I_{HP_n}}{P_m - P_n} = \sum_{\nu} \frac{G_\nu \bar{B}_{\nu P_{mn}}}{R_{H\nu}} J_{H\nu}. \tag{11}$$

If we use this expression, however, we shall make use of only the observations at P_m and P_n, so that our intermediate observations would contribute nothing to our accuracy. For this reason, the plan adopted was to obtain from the curve, or from the corresponding empirical expressions, the values of I_{HP} corresponding to P = 100, 200, 300 $\cdots$ 1,000 lbs. per sq. inch. The observations were then divided into pressures, 100 to 500 lbs., and the others to pressures, 600 to 1,000 lbs. inc. Denoting the 100 lb. pressure by P_1, the 200 lb. pressure by P_2, and so on, we then have, from (11),

$$\begin{aligned}
\frac{I_{HP_4} - I_{HP_1}}{500} &= \sum_\nu \frac{G_\nu \bar{B}_{\nu P_{4,1}}}{R_{H\nu}} J_{H\nu}, \\
\frac{I_{HP_7} - I_{HP_2}}{500} &= \sum_\nu \frac{G_\nu \bar{B}_{\nu P_{7,2}}}{R_{H\nu}} J_{H\nu}, \\
&\cdots\cdots\cdots \\
\frac{I_{HP_{10}} - I_{HP_5}}{500} &= \sum_\nu \frac{G_\nu \bar{B}_{\nu P_{10,5}}}{R_{H\nu}} J_{H\nu}.
\end{aligned} \tag{12}$$

Each of these equations corresponds to a separate pair of observations; and, by taking the mean of the left hand sides and of the right hand sides, we get for the assigned value of H, an equation controlled by the ten individual observations. Thus, writing the average of the difference $I_{HP_m} - I_{HP_n}$ as ΔI_H, and writing $\bar{B}_\nu$ as the average of all the ten $\bar{B}_{\nu P_{m,n}}$, we have

$$\Delta I_H = 500 \sum_\nu \frac{G_\nu \bar{B}_\nu}{R_{H\nu}} J_{H\nu}. \tag{13}$$

Since $\bar{B}_\nu$ is the same for all the altitudes concerned, we may write

$$500\, \bar{G}_\nu \bar{B}_\nu \equiv A_\nu,$$

where A_ν is independent of the altitude, and involves only ν and the magnitude of the pressure range $P_{m,n}$. Thus,

$$\Delta I_H = \sum_\nu \frac{A_\nu}{R_{H\nu}} J_{H\nu}. \tag{14}$$

Remembering that $J_{H\nu}$ involves ν only in the coefficients μ_ν and that A_ν and $R_{H\nu}$ are functions of μ_ν, since the coefficient of absorption is a function of ν,[9] it is now our business to assign a number of values of μ_ν, calculate the $R_{H\nu}$ in the manner already described, then calculate the $J_{H\nu}$ for each altitude, and use the result for the three altitudes in the determination of the A_ν's.

It might seem that we could always fit the experimental results by the assignment of any *three* μ_ν's since if there were

[9] We desire to emphasize the fact that it is only necessary to assign the μ_ν and not the ν's, so that our adjustment of the μ_ν's to the data does not involve knowledge of the form of dependence of μ_ν on ν.

only three terms in the sum, we could determine from our three equations for the three altitudes, the three values of A_r. If, however, we attempt this procedure, we find that in general it will be defeated by one or more of the A_r's, coming out negative. The result would correspond to an impossible situation. It would correspond to the entry into our atmosphere of a "preventer of the creation of ions," whose "power to prevent" fell off according to the same law as that for the ionizing power of the other radiations.

Table XIV represents, for different values of H, the values of I_{HP} of Equation (12), for the values of P given in the second column. The Table also gives the values of $I_{HP_m} - I_{HP_n}$ and the mean value ΔI_H for each value of H.

TABLE XIV.

Location and Value of H.	P.	I_{HP}.	$I_{HP_m} - I_{HP_n}$.	ΔI_H.
Pike's Peak, H = 14,000 ft.	100	38.5		44.2
	200	61.9		
	300	78.4		
	400	90.6		
	500	100	$I_{H6} - I_{H1} = 68.8$	
	600	107	$I_{H7} - I_{H2} = 51.9$	
	700	114	$I_{H8} - I_{H3} = 40.6$	
	800	119	$I_{H9} - I_{H4} = 32.9$	
	900	123	$I_{H10} - I_{H5} = 26.9$	
	1,000	127		
Colorado Springs, H = 6,000 ft.	100	17.4		20.9
	200	26.1		
	300	33.6		
	400	40.1		
	500	44.2	$I_{H6} - I_{H1} = 30.7 = 68.8$ (0.446)	
	600	48.1	$I_{H7} - I_{H2} = 25.0 = 51.9$ (0.482)	
	700	51.1	$I_{H8} - I_{H3} = 20.1 = 40.6$ (0.495)	
	800	53.7	$I_{H9} - I_{H4} = 15.5 = 32.9$ (0.471)	
	900	55.6	$I_{H10} - I_{H5} = 13.1 = 26.9$ (0.487)	
	1,000	57.3		
New Haven, Sea Level	100	11.4		14.8
	200	17.5		
	300	22.5		
	400	26.6		
	500	29.9	$I_{H6} - I_{H1} = 21.2 = 68.8$ (0.308)	
	600	32.6	$I_{H7} - I_{H2} = 17.4 = 51.9$ (0.335)	
	700	34.9	$I_{H8} - I_{H3} = 14.2 = 40.6$ (0.350)	
	800	36.7	$I_{H9} - I_{H4} = 11.6 = 32.9$ (0.353)	
	900	38.2	$I_{H10} - I_{H5} = 9.6 = 26.9$ (0.357)	
	1,000	39.5		

A Subsidiary Point of Interest. It will be observed that the ratios of $I_{HP_m} - I_{HP_n}$ for the three altitudes come out sensibly the same for the fixed value of $P_m - P_n$. This is shown by the equality of the numbers in parentheses for a given station. A similar result holds if we evaluate dI_{HP}/dP for the pressures 100, 200, $\cdots$ etc. It turns out that the ratios of the dI_{HP}/dP for the three altitudes and a fixed value of P are independent of that value. This is shown more explicitly in Table XV, where again, the approximate equality of the numbers in parenthesis in the vertical column, supports the statement.

TABLE XV.

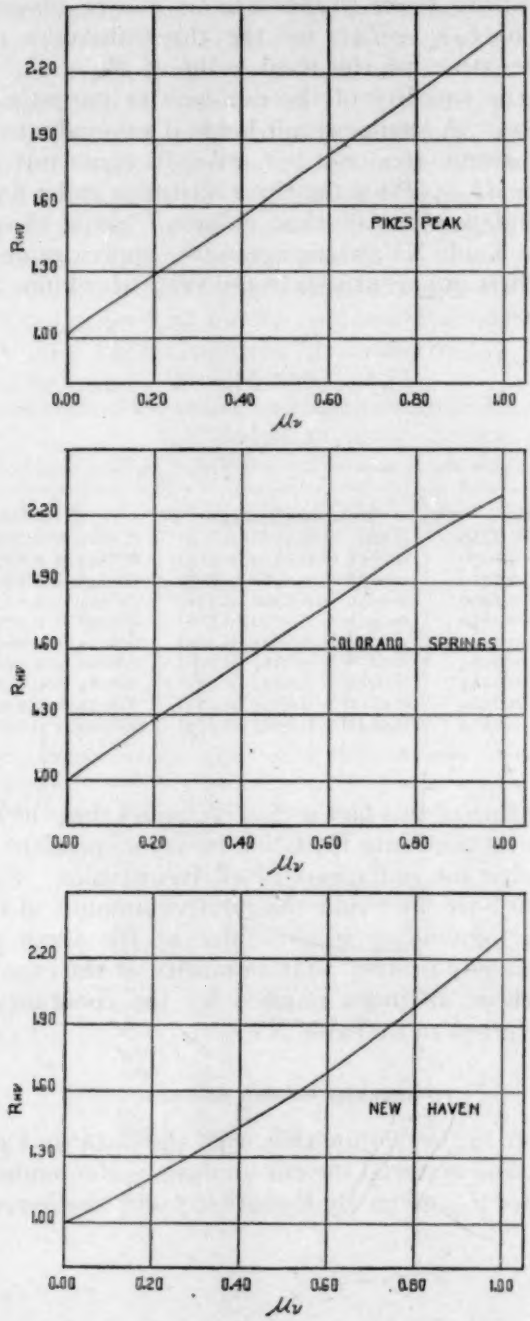
P Lbs./inch.^{2}	$\frac{dI_H}{dP}$.		
	Pike's Peak	Colorado Springs	New Haven
100	0.2780	0.101 = 0.2780 (0.363)	0.0664 = 0.2780 (0.239)
200	0.1970	0.0815 = 0.1970 (0.414)	0.0543 = 0.1970 (0.276)
300	0.1410	0.0658 = 0.1410 (0.467)	0.0445 = 0.1410 (0.316)
400	0.1200	0.0530 = 0.1200 (0.442)	0.0364 = 0.1200 (0.303)
500	0.0830	0.0428 = 0.0830 (0.516)	0.0298 = 0.0830 (0.359)
600	0.0695	0.0344 = 0.0695 (0.495)	0.0244 = 0.0695 (0.351)
700	0.0584	0.0278 = 0.0584 (0.476)	0.0200 = 0.0584 (0.342)
800	0.0484	0.0223 = 0.0484 (0.461)	0.0164 = 0.0484 (0.339)
900	0.0394	0.0181 = 0.0394 (0.459)	0.0134 = 0.0394 (0.340)
1,000	0.0303	0.0145 = 0.0303 (0.479)	0.0109 = 0.0303 (0.360)

The significance of this fact is that it implies that the quantity $B_{\nu P}$, first introduced into Equation (9) is independent of ν, i.e., it is the same for radiations of all frequencies. For if $B_{\nu P}$ depended on ν the fact that the relative amount of radiation at different frequencies would differ at the three altitudes would necessarily destroy that similarity of the experimental curve for those altitudes implied by the constancy of the numbers referred to in Table XV.

ANALYSIS OF THE RESULTS.

Equation 14, in conjunction with the data in Table XIV forms the basic material for our analysis. Remembering the expression for $J_{H\nu}$ given by Equation 5 and the curve, Fig. 5,

FIG. 5.


2.20
1.90
1.60
1.30
1.00
R_{HF}
PIKES PEAK
0.00
0.20
0.40
0.60
0.80
1.00
μ_v
2.20
1.90
1.60
1.30
1.00
R_{HF}
COLORADO SPRINGS
0.00
0.20
0.40
0.60
0.80
1.00
μ_v
2.20
1.90
1.60
1.30
1.00
R_{HF}
NEW HAVEN
0.00
0.20
0.40
0.60
0.80
1.00
μ_v

for $R_{H\nu}$ as a function of μ_ν for each altitude, it will be seen that we can readily construct $J_{H\nu}/R_{H\nu}$ as a function of μ_ν for each of the three values of H_ν. The relationship is exhibited in Table XVI, which forms a convenient basis for the interpretation of the results of the experiment in relation to the absorption coefficient.

TABLE XVI.

$\frac{J_{H\nu}}{R_{H\nu}} \times 10^3$.				$\frac{J_{H\nu}}{R_{H\nu}} \times 10^3$.			
μ_ν.	Pike's Peak.	Colorado Springs.	New Haven.	μ_ν.	Pike's Peak.	Colorado Springs.	New Haven.
0.00	1,000	1,000	1,000				
0.01	755	717	713	0.52	4.95	1.15	0.340
0.02	633	582	555	0.54	4.19	0.950	0.259
0.04	473	407	364	0.56	3.52	0.825	0.211
0.06	367	295	250	0.58	3.02	0.685	0.171
0.08	288	220	175	0.60	2.56	0.550	0.128
0.10	230	165	124	0.62	2.17	0.416	0.0995
0.12	183	125	92.5	0.64	1.85	0.330	0.0780
0.14	146	96.0	66.0	0.66	1.58	0.258	0.0599
0.16	121	75.2	48.5	0.68	1.33	0.217	0.0472
0.18	99.0	58.4	37.0	0.70	1.13	0.185	0.0375
0.20	81.5	45.3	27.5	0.72	0.985	0.157	0.0295
0.22	66.5	35.7	19.6	0.74	0.840	0.133	0.0232
0.24	55.6	28.1	15.1	0.76	0.720	0.112	0.0182
0.26	46.5	22.3	11.3	0.78	0.619	0.0940	0.0140
0.28	39.0	18.0	8.60	0.80	0.520	0.077	0.011
0.30	32.3	14.1	6.45	0.82	0.44	0.063	0.0081
0.32	27.3	11.3	4.85	0.84	0.36	0.052	0.0060
0.34	22.8	9.10	3.75	0.86	0.29	0.040	0.0048
0.36	19.2	7.25	2.90	0.88	0.25	0.033	0.0039
0.38	16.2	5.85	2.20	0.90	0.21	0.025	0.0031
0.40	13.5	4.60	1.65	0.92	0.18	0.021	0.0025
0.42	11.5	3.73	1.31	0.94	0.16	0.017	0.0019
0.44	9.50	3.00	1.02	0.96	0.14	0.013	0.0015
0.46	8.10	2.46	0.785	0.98	0.12	0.010	0.0011
0.48	6.88	1.96	0.595	1.00	0.10	0.0080	0.0007
0.50	5.80	1.50	0.450				

For convenience we shall write

$$\frac{J_{H\nu}}{R_{H\nu}} \equiv K_{H\nu}, \tag{15}$$

so that (14) becomes

$$\Delta I_H = \Sigma_\nu A_\nu K_{H\nu}. \tag{16}$$

Possibilities Inherent in the Assumption of Radiation of a Single Frequency. Our first thought is naturally to try the

possibilities following from the assumption of a single coefficient of absorption, so that the summation in Equation (16) would be represented by a single term. Then, if H_1 and H_2 correspond to Pike's Peak and Colorado Springs respectively, we have, from the observations of these stations

$$\frac{\Delta I_{H_1\nu}}{\Delta I_{H_2\nu}} = \frac{K_{H_1\nu}}{K_{H_2\nu}}. \tag{17}$$

From the data in Table XIV

$$\frac{\Delta I_{H_1\nu}}{\Delta I_{H_2\nu}} = 2.11. \tag{18}$$

It is now readily possible to find by interpolation, from Table XVI, the value of μ_ν for which this ratio is realized. We find 0.265, for the value of μ_ν in question. Using this value in the Equation

$$\Delta I_{H_1\nu} = A_\nu K_{H_1\nu}, \tag{19}$$

we find

$$A_\nu = 993.$$

Using these values of A_ν and μ_ν for the calculation ΔI_H for New Haven we find the value 10.5 which differs from the value recorded in Table XIV by 29.1 per cent., which represents a discrepancy quite inadmissible in view of the accuracy of the experiment. We thus conclude that it is impossible to explain the data by the assumption of the single coefficient of absorption.

Possibilities Inherent in the Assumption of Two Frequencies. Bearing in mind the experimental data in Table XIV, Equation (16) now gives for the three values of ΔI_H

$$\Delta I_{H_1} = A_{\nu_1} K_{H_1\nu_1} + A_{\nu_2} K_{H_1\nu_2}, \tag{20}$$

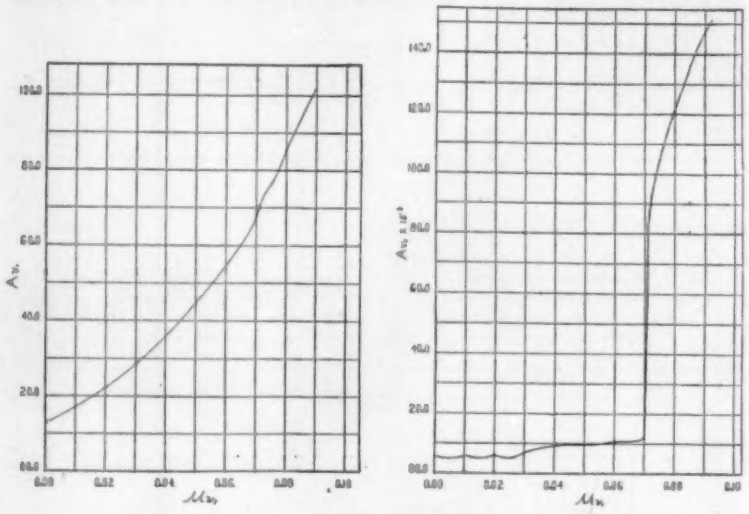
$$\Delta I_{H_2} = A_{\nu_1} K_{H_2\nu_1} + A^{\nu}{}_2 K_{H_2\nu_2}, \tag{21}$$

$$\Delta I_{H_3} = A_{\nu_1} K_{H_3\nu_1} + A_{\nu_2} K_{H_3\nu_2}. \tag{22}$$

There are four unknowns in these equations, namely, A_{ν_1}, A_{ν_2}, μ_{ν_1} and μ_{ν_2}. It is consequently possible to solve the equations for A_{ν_1}, A_{ν_2}, μ_{ν_2}, in terms of μ_{ν_1}. When this is done, and the

results are plotted against μ_{ν_1}, Figs. 6 and 7 are obtained,[10] in which, in Fig. 6, one curve refers to A_{ν_1}, and the other to A_{ν_2}. Fig. 8 is a more detailed picture of one branch of Fig. 7.

FIG. 6.

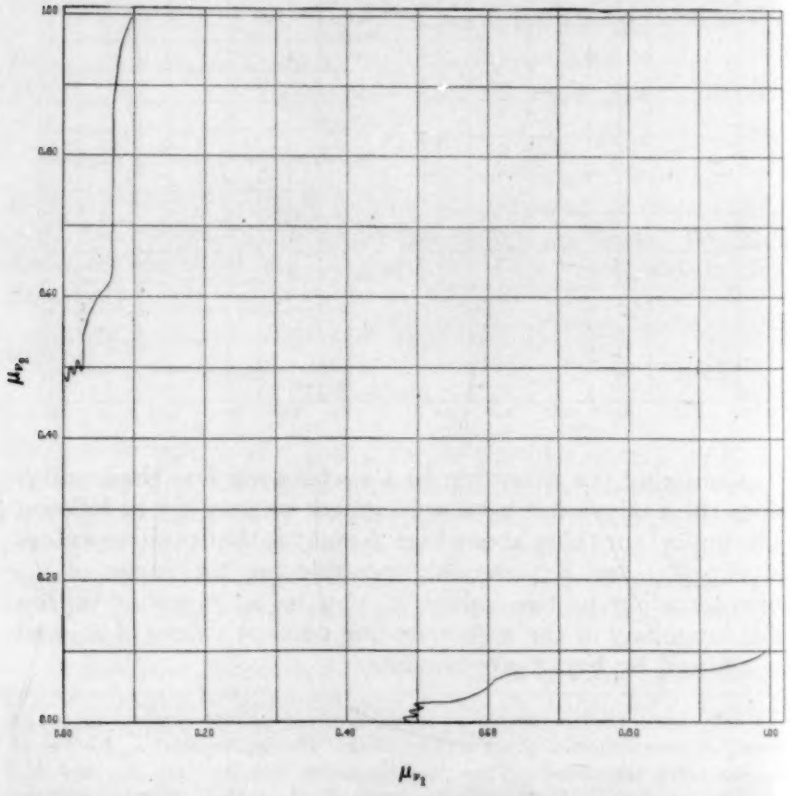


Confining our attention to μ's which are less than unity, since the analysis has been so confined, we may say as follows: The important thing about Figs. 6 and 7 is that positive values of A_{ν_1}, A_{ν_2}, and μ_{ν_2}, are only possible for the range of μ_{ν_1}, corresponding to the values $\mu_{\nu_1} = 0$ to $\mu_{\nu_1} = 0.09$. Within this range any of the corresponding pairs of values of μ_{ν_1}, and μ_{ν_2} allowed by Fig. 7, are possible.

[10] In view of the fact that the μ's are not involved in simple algebraic form, the procedure here indicated is not without labor. Having assumed μ_{ν_1}, a value of μ_{ν_2} was taken tentatively. Then, from Equation (20) and (21), A_{ν_1}, and A_{ν_2}, could be calculated. These tentative values A_{ν_1}, A_{ν_2}, and μ_{ν_2}, together with the assigned value of μ_{ν_1}, were then substituted in Equation (22). In general, the equation was not satisfied. The difference, ϵ, between the two sides was calculated. The procedure was then repeated with the same value of μ_{ν_1}, but with another tentative value of μ_{ν_2}, and the value of ϵ was determined again. Proceeding in this way, it was possible to determine a number of values of ϵ corresponding to the various tentatively chosen values of μ_{ν_2}, and to plot the ϵ's against the μ_{ν_2}'s. The value of μ_{ν_2} for which ϵ became zero was then the value appropriate to the solution of all three equations (20), (21), and (22).

One branch of the curve becomes asymptotic to a line parallel to the axis of μ_{ν_1} at $\mu_{\nu_2} = 0.135$ and, of course, the other branch becomes asymptotic to a line parallel to the μ_{ν_2} axis at $\mu_{\nu_1} = 0.135$. The process of finding the value 0.135 is as follows: If the curve becomes asymptotic to a line parallel

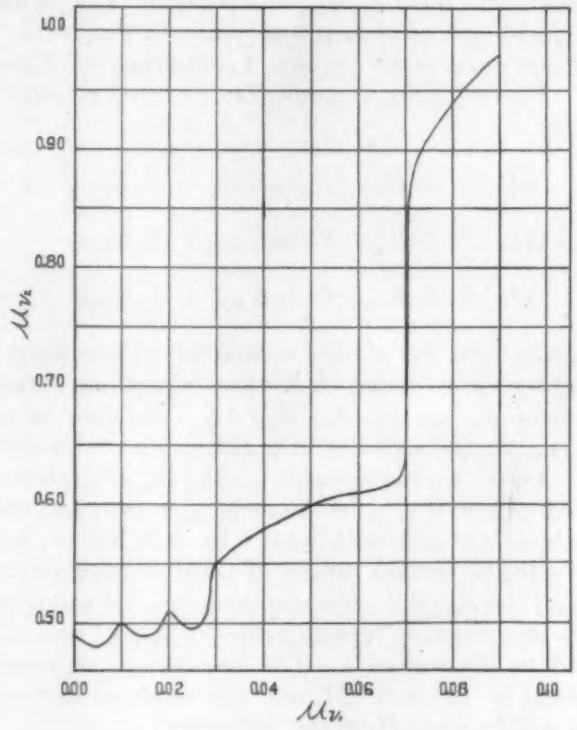
FIG. 7.



to the μ_{ν_2} axis at some value $\mu_{\nu_1} = a$, then we have a two μ solution with $\mu_{\nu_1} = a$, $\mu_{\nu_2} = \infty$. Now it is possible to find a single μ solution of the type 19, which will fit the experimental results for any two altitudes, those of Colorado Springs and New Haven for example. If this curve of $\Delta I_{H\nu}$ plotted against H falls below the value of $\Delta I_{H\nu}$ for Pike's Peak, as in fact it does, it is of course possible to choose another μ infinite

in value, say μ_∞ and associate it with a new value of A_ν, say A_{ν_∞} of such an order that $A_{\nu_\infty}K_{H\nu_\infty}$ is just equal to the excess of the actual value of $\Delta I_{H\nu}$ for Pike's Peak over the value

FIG. 8.



corresponding to the single term $A_a K_{Ha}$ referred to above. The expression

$$\Delta I_H = A_{\nu_\infty}K_{H\nu_\infty} + A_a K_{Ha}$$

will then fit the observations for Pike's Peak, Colorado Springs and New Haven. For, on account of the infinite $\nu = \nu_\infty$ the term $A_{\nu_\infty}K_{H\nu_\infty}$ will be zero at Colorado Springs and New Haven if it is finite at Pike's Peak. The net result of all this is that the value $\mu_{\nu_1} = a$ at which μ_{ν_2} becomes infinite is given by the value of μ_ν which fits the two observations for Colorado

Springs and New Haven, respectively. It is of importance to observe that the value $\mu_{\nu_1} = 0.135$ is less than the value 0.48 at which the second branch of the curve starts in.

A second feature of these curves may be remarked upon. There is a solution for $\mu_{\nu_1} = 0$ and $\mu_{\nu_2} = 0.48$. A value $\mu_{\nu_1} = 0$ makes the term $A_{\nu_1}K_{H\nu_1}$ independent of H, as indeed is implied by the lack of absorption typified by $\mu_{\nu_1} = 0$.

Possibilities Inherent in the Assumption of Three Frequencies. The equations applicable to the assumption of three μ's are:

$$\Delta I_{H_1} = A_{\nu_1}K_{H_1\nu_1} + A_{\nu_2}K_{H_1\nu_2} + A_{\nu_3}K_{H_1\nu_3}, \tag{23}$$

$$\Delta I_{H_2} = A_{\nu_1}K_{H\nu_1} + A_{\nu_2}K_{H_2\nu_2} + A_{\nu_3}K_{H_2\nu_3}, \tag{24}$$

$$\Delta I_{H_3} = A_{\nu_1}K_{H_3\nu_1} + A_{\nu_2}K_{H_3\nu_2} + A_{\nu_3}K_{H_3\nu_3}. \tag{25}$$

Remembering that the μ's are completely determined by the corresponding ν's, we have, thus, three equations between the six unknowns μ_{ν_1}, μ_{ν_2}, μ_{ν_3}, A_{ν_1}, A_{ν_2}, A_{ν_3}. We can, in general, assign μ_{ν_1}, μ_{ν_2}, μ_{ν_3} and calculate the three A's which will give a solution. Thus, we may assign μ_{ν_1} and μ_{ν_2} and then take a whole set of different μ_{ν_3}'s for the same μ_{ν_1} and μ_{ν_2} and calculate the A's which will give solutions. In such a case, however, there will only be certain ranges of possible assignment of μ_{ν_3} for which all the A's will come out positive. It is our problem to specify the possible sets of values of μ_{ν_1}, μ_{ν_2} and μ_{ν_3} which correspond to positive A's. The results are represented in tabular form in Table XVII, and the mode of derivation of this table will be clear from the following:

In the first place, having assigned three μ's it is obvious that an algebraical solution of (23)–(25) for the A's can only give positive, negative, or zero values. It cannot give imaginary or complex values. Again, it is obvious that in the case where three μ's are under consideration, the relation between μ_{ν_2} and μ_{ν_1} for which there is a two μ solution with A_{ν_3} zero is the same as the relation between μ_{ν_2} and μ_{ν_3} for which there is a two μ solution with A_{ν_1} zero. In other words, Fig. 7 will serve for both cases, the horizontal axis being labelled μ_{ν_1} in one case and μ_{ν_3} in the other.

TABLE XVII.

μ_{P_2} = 0.01.		μ_{P_2} = 0.05.		μ_{P_2} = 0.08.		
μ_{P_1}.	μ_{P_2}.	μ_{P_1}.	μ_{P_2}.	μ_{P_1}.	μ_{P_2}.	
0.02	0.50 → 0.51	0.02	0.51 → 0.60	0.02	0.51 → 0.94	
0.04	0.50 → 0.58	0.04	0.58 → 0.60	0.04	0.58 → 0.94	
0.06	0.50 → 0.61	0.06	0.61 → 0.60	0.06	0.61 → 0.94	
0.08	0.50 → 0.94	0.08	0.94 → 0.60	0.08	***	
0.10	0.50 → (> 0.98 < ∞)	0.10	(> 0.98 < ∞) → 0.60	0.10	(> 0.98 < ∞) → 0.94	
0.15	0.50 → ∞	0.15	∞ → 0.60	0.15	∞ → 0.94	
0.20	0.50 → ∞	0.20	∞ → 0.60	0.20	∞ → 0.94	
0.25	0.50 → ∞	0.25	∞ → 0.60	0.25	∞ → 0.94	
0.30	0.50 → ∞	0.30	∞ → 0.60	0.30	∞ → 0.94	
0.35	0.50 → ∞	0.35	∞ → 0.60	0.35	∞ → 0.94	
0.40	0.50 → ∞	0.40	∞ → 0.60	0.40	∞ → 0.94	
0.45	0.50 → ∞	0.45	∞ → 0.60	0.45	∞ → 0.94	
0.50	**	0.50	∞ → 0.60	0.50	∞ → 0.94	0.00 → 0.01
0.55	0.50 → 0.03	0.55	∞ → 0.60	0.55	∞ → 0.94	0.00 → 0.03
0.60	0.50 → 0.05	0.60	**	0.60	∞ → 0.94	0.00 → 0.05
0.65	0.50 → 0.07	0.65	0.07 → 0.60	0.65	∞ → 0.94	0.00 → 0.07
0.70	0.50 → 0.07	0.70	0.07 → 0.60	0.70	∞ → 0.94	0.00 → 0.07
0.75	0.50 → 0.07	0.75	0.07 → 0.60	0.75	∞ → 0.94	0.00 → 0.07
0.80	0.50 → 0.07	0.80	0.07 → 0.60	0.80	∞ → 0.94	0.00 → 0.07
0.85	0.50 → 0.07	0.85	0.07 → 0.60	0.85	∞ → 0.94	0.00 → 0.07
0.90	0.50 → 0.074	0.90	0.074 → 0.60	0.90	∞ → 0.94	0.00 → 0.074
0.95	0.50 → 0.082	0.95	0.082 → 0.60	0.95	0.082 → 0.94	

TABLE XVII.—*Continued.*

μ_{ν_2} = 0.135 → 0.48.		μ_{ν_2} = 0.60.		μ_{ν_2} = 0.90.	
μ_{ν_1}.	μ_{ν_2}.	μ_{ν_1}.	μ_{ν_2}.	μ_{ν_1}.	μ_{ν_2}.
0.02	0.51 → ∞	0.02	0.05 → 0.51	0.02	0.074 → 0.51
0.04	0.58 → ∞	0.04	0.05 → 0.58	0.04	0.074 → 0.58
0.06	0.61 → ∞	0.06	0.05 → 0.00	0.06	0.074 → 0.61
0.08	0.94 → ∞	0.08	0.05 → 0.00 0.94 → ∞	0.08	0.074 → 0.00 0.94 → ∞
0.10	> 0.98 < ∞ → ∞	0.10	0.05 → 0.00 (> 0.98 < ∞) → ∞	0.10	0.074 → 0.00 (> 0.98 < ∞) → ∞
0.15	*	0.15	0.05 → 0.00	0.15	0.074 → 0.00
0.20	*	0.20	0.05 → 0.00	0.20	0.074 → 0.00
0.25	*	0.25	0.05 → 0.00	0.25	0.074 → 0.00
0.30	*	0.30	0.05 → 0.00	0.30	0.074 → 0.00
0.35	*	0.35	0.05 → 0.00	0.35	0.074 → 0.00
0.40	*	0.40	0.05 → 0.00	0.40	0.074 → 0.00
0.45	*	0.45	0.05 → 0.00	0.45	0.074 → 0.00
0.50	0.00 → 0.025	0.50	0.05 → 0.01	0.50	0.074 → 0.01
0.55	0.00 → 0.03	0.55	0.05 → 0.03	0.55	0.074 → 0.03
0.60	0.00 → 0.05	0.60	***	0.60	0.074 → 0.05
0.65	0.00 → 0.07	0.65	0.05 → 0.07	0.65	0.074 → 0.07
0.70	0.00 → 0.07	0.70	0.05 → 0.07	0.70	0.074 → 0.07
0.75	0.00 → 0.07	0.75	0.05 → 0.07	0.75	0.074 → 0.07
0.80	0.00 → 0.07	0.80	0.05 → 0.07	0.80	0.074 → 0.07
0.85	0.00 → 0.07	0.85	0.05 → 0.07	0.85	0.074 → 0.07
0.90	0.00 → 0.074	0.90	0.05 → 0.074	0.90	***
0.95	0.00 → 0.082	0.95	0.05 → 0.082	0.95	0.074 → 0.082

Regarding the above table, the following remarks should be made.

1. The expression ($> 0.98 < \infty$) associated with $\mu_{\nu_3} = 0.01$, $\mu_{\nu_1} = 0.10$ is intended to indicate that the upper limit of the μ_{ν_2} range is less than infinity, but is above the highest value 0.98 for which the curve in Fig. 7 has been drawn. This notation and similar ones in other parts of the table, is used to distinguish the range from one which extends completely to infinity, as for example as in the case $\mu_{\nu_3} = 0.01$, and $\mu_{\nu_1} = 0.20$.

2. It will be noted that in certain cases there is a double range for μ_{ν_1}, as for example, in the case corresponding to $\mu_{\nu_3} = 0.60$, and $\mu_{\nu_1} = 0.08$.

3. The * indicates that for the values of μ_{ν_3} and μ_{ν_1} cited, there is no value of μ_{ν_2} which would give a solution with positive A's.

4. The ** indicates a case where there is a two μ solution with μ_{ν_1} and μ_{ν_3}. It turns out that whenever such a situation occurs, there is always an infinite number of three μ solutions corresponding in general, to one definite value of μ_{ν_2}. There is no range of μ_{ν_2} corresponding to this condition, however, and the values of the A's are indeterminate. One can assign any value of A_{ν_1} he chooses, and determine corresponding values of A_{ν_3}, and A_{ν_2}. It has not been thought worthwhile to investigate the particular isolated values of μ_{ν_2} for which the few solutions occur in these cases, nor to test the sign of the possible A's associated with them in order to see if they are always positive or whether the three μ solutions can, in this case, have all three A's positive everywhere.

5. The figures marked ***, correspond to a more or less trivial case. For them, $\mu_{\nu_1} = \mu_{\nu_3}$, so that A_{ν_1} and A_{ν_3} combine together to a single A. There is of course no range for μ_{ν_2} but only a single value corresponding to the equal values of μ_{ν_1} and μ_{ν_3}, and that single value, or, at most, a few isolated values, corresponding to cases where the curve is multivalued can be read off from the curve, Fig. 7.

A typical case illustrative of the results recorded in Table XVII, may be represented symbolically as follows:

	$\mu_{\nu_3} = \alpha$
μ_{ν_1}	μ_{ν_2}
β	$p \rightarrow q$

It corresponds to the statement that for $\mu_{\nu_3} = \alpha$, and $\mu_{\nu_1} = \beta$, a solution with positive A's may be found for any value of μ_{ν_2} from p to q. The following will indicate how the limits p and q are obtained: referring to Fig. 7, let the horizontal axis denote μ_{ν_3} and the vertical axis μ_{ν_2}. Then, for $\mu_{\nu_3} = \alpha$, we find a value $\mu_{\nu_2} = p$ corresponding to a two μ solution with finite A_{ν_2} and A_{ν_3}, and consequently with zero A_{ν_1}. We shall discuss separately the case where the curve gives more than

one value of μ_{ν_2} for a given value of μ_{ν_3}. Now let us use Fig. 7 again, with the horizontal axis corresponding to μ_{ν_1}, and let us find the value $\mu_{\nu_3} = q$ corresponding to $\mu_{\nu_1} = \beta$, for which there is a two μ solution with finite A_{ν_1}, and A_{ν_3}, and consequently with $A_{\nu_2} = 0$. Suppose now we can find any value $\mu_{\nu_2} = \gamma$ within the range p to q, such that for $\mu_{\nu_3} = \alpha$, $\mu_{\nu_1} = \beta$, $\mu_{\nu_2} = \gamma$, all of the A's are positive. Then, we shall prove that the A's are all positive for $\mu_{\nu_3} = \alpha$, $\mu_{\nu_1} = \beta$, for the whole range $\mu_{\nu_2} = p$ to q. For suppose, keeping μ_{ν_3} and μ_{ν_1} fixed, we change μ_{ν_2} from the value γ, the only way in which any of the A's could become negative is (1) by passing through zero, or (2) by mounting to infinity and becoming discontinuous. Case (1) is excluded because if one of the A's became zero at say $\mu_{\nu_2} = \lambda$, we should have a two μ solution with $\mu_{\nu_2} = \lambda$ and one of the other μ's either $\mu_{\nu_1} = \beta$ or $\mu_{\nu_3} = \alpha$. But, the only two μ solutions with $\mu_{\nu_1} = \beta$ and $\mu_{\nu_3} = \alpha$ occur at $\mu_{\nu_2} = q$ and $\mu_{\nu_2} = p$ respectively. Case (2) is excluded for the following reason: suppose that, starting from $\mu_{\nu_2} = \lambda$ we move away from that value, keeping the other μ's constant, and suppose that one of the A's, say A_s is the first to become infinite; or, let us say that none of the A's become infinite before A_s. Then, if A_s becomes infinite at $\mu_{\nu_2} = k$, we shall have, at $\mu_{\nu_2} = k$, $A_s = \infty$, and the other A's positive, if indeed they are not infinite. Such a condition is impossible since all the ΔI_H's in equations (23)–(25) are finite.

If one of the end points p or q is infinite, or even if there is only one value of μ_{ν_2} between zero and infinity for which there is a two μ solution; i.e., if even $\mu_{\nu_2} = \infty$ does not give a two μ solution, the argument is not altered; for the only use that we made of the end points p and q was their property that there was no value of μ_{ν_2} between them giving a two μ solution.

To a point in the above argument there is a *corollary* which is useful in discussing the ranges. It is to the following effect: suppose that, for $\mu_{\nu_3} = \alpha$, $\mu_{\nu_1} = \beta_1$ and $\mu_{\nu_2} = \gamma$, there is a solution with all the A's, finite and positive. Then, all of the A's will be finite and positive for $\mu_{\nu_3} = \alpha_1$, $\mu_{\nu_1} = \beta_1$ and $\mu_{\nu_2} = \gamma_1$, provided that if

α_0 is *any* μ_{ν_3} in the range α to α_1,
β_0 is *any* μ_{ν_1} in the range β to β_1,
γ_0 is *any* μ_{ν_2} in the range γ to γ_1,

no pair of the quantities α_0, β_0, and γ_0 form a two μ solution. The proof is as follows: suppose that starting with any of the μ's, say μ_{ν_3} we change the value from α in the direction towards α_1. None of the A's can become negative by passing through zero; for, if one of the A's became zero at $\mu_{\nu_3} = \alpha_0$ there would be a two μ solution with $\mu_{\nu_3} = \alpha_0$ and one of the other μ's, $\mu_{\nu_1} = \beta$ or $\mu_{\nu_2} = \gamma$. Nor can any of the A's become negative by passing through infinity. For, if A_ϵ is one which becomes infinite first, or no later than any of the others, and if this happens at $\mu_{\nu_3} = \alpha_0$, then, for $\mu_{\nu_3} = \alpha_0$, $\mu_{\nu_1} = \beta$, $\mu_{\nu_2} = \gamma$, we shall have all the A's positive, and one, at least, infinite. Such a case is inconsistent with a finite ΔI_H in accordance with equations (23)–(25). Having shifted μ_{ν_3} from α to α_1, with the condition that there is no two μ solutions with any pair of α_0, β, γ, where α_0 lies between α and α_1, we may then shift β to β_1, with the condition that there is no two μ solutions with any pair of β_0, α_1, γ where β_0 lies between β and β_1. Having done this, we may shift γ to γ_1 with the condition that there is no two μ solution with any pair of γ_0, α, β_1, where γ_0 lies between γ and γ_1.

Since the ranges of α_0, β_0 and γ_0 include the end values α and α_1, β and β_1, γ and γ_1, respectively, all of the above conditions are included in the condition that there shall be no two μ solutions for any pair of α_0, β_0, γ_0 with the ranges specified. Indeed, this latter condition is more than sufficient. The foregoing corollary enables us to extend one set of values $\mu_{\nu_3} = \alpha$, $\mu_{\nu_1} = \beta$, $\mu_{\nu_2} = \gamma$ for which there are positive A's, to a whole range of cases, such for example, as those occurring in the table for $\mu_{\nu_3} = 0.00$ to 0.05, $\mu_{\nu_1} = 0.135$ to 0.48, $\mu_{\nu_2} = 0.60$ to ∞.

If now we consider any set of μ's in which μ_{ν_2} falls outside of the ranges p to q discussed above, we can show that if the A's are not all positive at any one point of the exterior ranges in question, they will not all be positive at any other point in it. For, suppose that one at least, of the A's, say A_ϵ is negative, at $\mu_{\nu_2} = \gamma$ in a range R of μ_{ν_2} for which there are no two μ solutions with $\mu_{\nu_3} = \alpha$ or $\mu_{\nu_1} = \beta$. Now if our theorem is not true there must be at least one value, say $\mu_{\nu_2} = h$ for which A_ϵ changes sign, for fixed values of μ_{ν_1} and μ_{ν_3}. None of the A's can change sign by passing through zero, since this would

involve a two μ solution at the point in question. Hence, A_ϵ must change sign by mounting first to $-\infty$, at $\mu_{\nu_2} = h$. Now, just after passing $\mu_{\nu_2} = h$, we have the following possibilities, in which A_η and A_ξ refer to the other two A's.

(1) $A_\epsilon = +\infty$, A_η = positive (finite or infinite), A_ξ = positive (finite or infinite).

(2) $A_\epsilon = +\infty$, A_η = negative (finite or infinite), A_ξ = positive (finite or infinite).

(3) $A_\epsilon = +\infty$, A_η = positive (finite or infinite), A_ξ = negative (finite or infinite).

(4) $A_\epsilon = +\infty$, A_η = negative (finite or infinite), A_ξ = negative (finite or infinite).

Case (1) is impossible, as may be seen from (23)–(25) bearing in mind that the ΔI_H are finite. All of the others leave us where we started, in the sense that we have a situation in which all of the A's are not positive. Thus the theorem is proved.

A corollary analogous to that proved above is applicable to the present case, and is to the following effect: Suppose that one or more of the A's are negative at $\mu_{\nu_3} = \alpha$, $\mu_{\nu_1} = \beta$, $\mu_{\nu_2} = \gamma$, then one at least, of them will be negative at any other set of values $\mu_{\nu_3} = \alpha_1$, $\mu_{\nu_1} = \beta_1$, $\mu_{\nu_2} = \gamma_1$ provided that if $\mu_{\nu_3} = \alpha_0$, $\mu_{\nu_1} = \beta_0$, $\mu_{\nu_2} = \gamma_0$ be any values between α and α_1, β and β_1, and γ and γ_1 respectively, a two μ solution cannot be constructed out of any pair of the quantities α_0, β_0, γ_0.

For, as in the last proof, suppose that one of the A's at least is negative. Let A_ϵ be the A in question, and suppose it should change sign at some point $\mu_{\nu_3} = \alpha_0$, $\mu_{\nu_1} = \beta$, $\mu_{\nu_2} = \gamma$. Then as in the last proof we can prove that the only possibilities consistent with finite values of ΔI_H leave us with at least one of the A's negative after passing through the value $\mu_{\nu_3} = \alpha_0$, provided of course, that there are no two μ solutions with α_0 and β or γ. Thus, at all points over the range α to α_1 one or more of the A's remain negative. Having shifted μ_{ν_3} from α to α_1, with the conditions that there are no two μ solutions with any point α_0, β, γ, where α_0 lies between α and α_1, we may then shift β to β_1 with the condition that there are no two μ solutions with any pair of β_0, α_1, γ, where β_0 lies between β and β_1. Having done this, we may shift γ to γ_1

with the condition that there are no two μ solutions with any pair of γ_0, α_1, β_1, where γ_0 lies between γ and γ_1. Thus, just as in the former corollary, we conclude that the condition for there being no two μ solutions with any pair of α_0, β_0, γ_0, within the range specified, is more than sufficient to provide for the truth of the theorem. It is by means of this corollary that the condition of inequality of sign of all of the A's for any one set of μ's may be extended to ranges of other sets in such a manner as to determine the regions excluded in Table XVII, as the regions for which one or more of the A's is negative.

So far, we have omitted consideration of cases where for one value of μ_{ν_2} (or of μ_{ν_1}) there is more than one value of μ_{ν_2} giving a two μ solution. However, no special case is really here involved; for, we may take any range R between successive values of μ_{ν_2} which correspond to two solutions, and, on the criteria already deduced, we may ascertain whether in that range all of the A's are positive or not. As a matter of fact, it turns out that in the multivalued region of the curve in Fig. 7, the individual ranges of μ_{ν_2} for which all of the A's are positive join up in the one continuous range.

SUMMARY.

Measurements have been made of the residual ionization in a closed vessel of steel one inch thick, shielded by a two inch casing of lead, over a range of pressure from atmospheric to one thousand pounds per sq. inch, at Pike's Peak (alt. 14,000 ft.), Colorado Springs (alt. 6,000 ft.), and New Haven (alt. 6,000 ft.). The ionization-pressure curves presents features of interest in relation to the interpretation of the processes associated with ionization in the vessel. However, in the present investigation they are used simply to obtain average relative value of the ionization due to the cosmic rays at the three altitudes concerned. A method of correcting the observations for absorption by the iron sphere, the lead case, and the walls of the buildings, has been developed. It is found that the data for the three stations can be correlated by the assumption of a pair of frequencies in the original cosmic radiation; and, as a matter of fact, an infinite number of such pairs are possible. They are represented in graphical form in Fig. 7. If we permit the assumption of three different

frequencies in the cosmic radiation, it is possible in general to assign two of the frequencies, and determine an infinite number of possibilities as regards the third, distributed over a definite range. The possibilities inherent in the assumption of three frequencies are contained in tabular form in Table XVII. Of course, the inclusion of observations from more than three altitudes would serve to fix more definitely the possibility as regards the analysis. However, the result of the analysis for the three stations concerned is possibly of interest for comparison with other data, and as an indication of the kind of information which it is possible to extract from observations of this kind. It is probable that the very large range of possibilities inherent in a solution with three coefficients of absorption will be rather surprising to anyone who considers a situation of this kind for the first time.

In addition to those whose assistance has already been acknowledged, I wish to acknowledge the assistance of Professor J. W. Broxon, who visited me while on Pike's Peak, and assisted in setting up the apparatus; and my thanks are due to Mr. C. A. Kotterman, and particularly to Mr. Andrew Longacre, and Mr. W. E. Ramsey for assistance in the reduction of the observations, and in the computational work.

PRECISION INDUSTRIAL RECORDERS AND CONTROLLERS.*

BY

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IN the brief period of this evening's meeting, it will not be possible to present a comprehensive survey of the designs and uses of industrial recorders and controllers. Neither will I bore you with lengthy descriptions of all of the details of the particular types of industrial recorders and controllers with which I happen to be most familiar. I shall attempt to outline trends in the development and use of industrial recorders and controllers, discussing details only as much as may be necessary to illustrate some of the general statements.

Later in discussing some of the fundamentals of design, it will be necessary, of course, to refer to some details.

TRENDS IN THE DEVELOPMENT AND USE OF INDUSTRIAL RECORDERS AND CONTROLLERS.

The Use of Fundamentally Correct Principles

The first trend to which I wish to call attention has to do with the increasing willingness on the part of industrial plant operators to make use of the proper instruments and apparatus necessary to give accurate and reliable results, even though this apparatus is comparatively complex in structure and comparatively high in cost. Throughout our important industries today there is evident a real appreciation of the need for accurate recording instruments and automatic controllers. During the entire development of industry, there always have been pioneers who appreciated the need for accurate measurement and control, and to whom we are indebted for much of the early progress along these lines, but the significant point is that in the early days, this appreciation was the unusual thing, while today it is the usual thing.

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Twenty (20) or 25 years ago many recording instruments which were sold with the claim that the instruments would improve the accuracy of control of a process proved to be less reliable than the judgment of the operator. Naturally, such results, and the skepticism on the part of the purchasers which followed, tended to retard the general application of industrial recorders. Gradually industrial recorders were improved, their use became more widespread, and eventually they gained a place of unquestionable standing in industry. Probably this same sort of thing could be said of the introduction of most automatic devices to industry.

One of the greatest handicaps under which the introduction of industrial recorders labored was the arbitrary dictum on the part of instrument manufacturers and users alike, that an industrial recorder must be simple and cheap. Naturally, as long as this dictum prevailed, success could not be expected in industrial applications of recorders where the fundamental requirements of the problem necessitated something neither simple nor cheap.

To illustrate this point, I should like to refer to temperature measurements in industry. I think that this is a fair example because if any industrial recorder is to be chosen as an example on account of the number in use or because of its general use in practically all industries, we must necessarily choose the industrial temperature recorder, particularly that type of industrial temperature recorder known as the recording pyrometer used for measuring the higher temperatures. Without giving reasons, I should like to state as a fact that since the introduction of industrial recording pyrometers up to and including the present time, the device known as a thermocouple has been, and is, the most satisfactory sensitive element for measuring the ordinary high temperatures. Accordingly, most of the industrial recording pyrometers have been designed to operate from thermocouples. As most of you probably know, the best thermocouples available when subjected to temperatures of 500° to 1,500° F. produce only a very small electromotive force or voltage, something like 15 thousandths of a volt at 500° and 50 thousandths of a volt at 1,500°. The task facing the industrial recording pyrometer is, therefore, that of recording accurately voltages in the order of

15 thousandths of a volt to 50 thousandths of a volt. The accuracy required at 1,000° F. is usually 5° F. or better, which means an accuracy of 2 ten-thousandths of a volt or better. Such small voltages can be expected to produce only very feeble currents in any ordinary circuit. Therefore, any industrial recording pyrometer operating from a thermocouple must operate reliably from these very feeble currents which are available. This is a fundamental consideration based on the laws of nature and cannot be altered by any arbitrary dictum on the part of instrument manufacturers or users. As long as nothing better than a thermocouple is discovered or invented for this use, or as long as no vastly better metals or alloys are available for making thermocouples, the problem of the instrument manufacturer is to design and build a recording pyrometer which will operate reliably from the feeble currents which the present types of thermocouples produce. The obstacles involved are by no means insurmountable, provided we are willing to face the fundamental requirements and are willing to ignore the dictum of simplicity and low price.

Without stopping at this time to explain the principles and the details of design of the instrument, I should like to make the statement that the recording potentiometer has been developed to meet satisfactorily the requirements of an industrial recorder for measuring accurately the small voltages produced by thermocouples used in high temperature measurements. While the recording potentiometer is neither extremely simple nor cheap, it has been generally accepted in industry because it gives accurately and reliably the information that is needed. I point this out as a striking example of the trend mentioned previously, that is, the trend toward the use in industrial recorders of fundamentally correct principles with little regard for the dictum of extreme simplicity and low price.

I do not want to leave you with the impression that the potentiometer recorder is the only satisfactory one for measuring temperature by means of thermocouples. The recording millivoltmeter is still used and has been much improved in recent years. However, I believe there is no longer any doubt that the potentiometer method is the ideal one for this purpose. This statement is in agreement with the opinion of the Bureau

of Standards as expressed in Technologic Paper No. 170 on the subject of "Pyrometric Practice." This publication states "The most accurate method for measuring the E.M.F. of a thermocouple is by the use of a potentiometer." The high accuracy of the potentiometer results from the fact that certain sources of error which must be carefully guarded against in the millivoltmeter are non-existant in the potentiometer. No one has a monopoly on the potentiometer circuit and there is good reason to believe that all manufacturers of high grade pyrometers will eventually use the potentiometer circuit in their recorders.

It may be interesting to you to digress for a moment in order to receive some facts indicating how generally the potentiometer has been accepted in industry. About sixteen years ago, when the recording potentiometer was first introduced to industry, the general principles of the potentiometer circuit were well understood, and potentiometers were generally used in various laboratory measurements. While I cannot find that any of the earlier textbooks specifically defined the potentiometer, and the standard cell used with it, as laboratory apparatus, these items were usually described along with other strictly laboratory apparatus in books that described them at all, and in books devoted to industrial instruments, frequently the potentiometer and its adjuncts were omitted entirely. These are indirect implications that the potentiometer and its adjuncts were considered then as strictly laboratory apparatus, and I have no doubt that if anyone familiar with the potentiometer at that time were asked to classify the potentiometer and its accessories as either laboratory apparatus or industrial plant apparatus, he would necessarily have had to classify them as laboratory apparatus. Today the situation is quite different. Let me give you some figures. At the present time, there are being manufactured in this country approximately 1,000 standard cells per month for regular sale. To anyone familiar only with laboratory uses of the potentiometer, this figure in itself is a surprising one. Perhaps more surprising, however, is the fact that over 90 per cent. of these cells are manufactured and sold for industrial plant use, primarily for use with the recording potentiometer; less than 100 of the total of 1,000 being manu-

factured and sold for laboratory purposes. As each potentiometer requires a standard cell, the figures for standard cells represent fairly well the figures for potentiometers. Accordingly, there can be no question that today the potentiometer is primarily an industrial plant instrument. I should not want you to conclude from this that the use of the potentiometers in laboratories is on the decline, for such is not the case. On the contrary, the use of potentiometers in laboratories is ever on the increase and the potentiometer in the laboratory is a more important piece of apparatus than ever before. Notwithstanding this, however, the very large use of the potentiometer and a standard cell in industrial plants unquestionably makes it necessary to classify the potentiometer and the standard cell as industrial plant apparatus.

The Use of Electrical Instruments for Non-Electrical Measurement and Control

I now wish to call attention to another important trend in the use of industrial recorders and controllers. I refer to the widespread use of electrical recording instruments and controllers for the measurement and control of non-electrical conditions and quantities. The following list, which is not intended to be complete, will give you some idea of the many non-electrical conditions and quantities of importance in industrial plant operation which are now being recorded with electrical instruments or automatically controlled by means of electrical measuring devices.

Temperature
Fluid flow
Liquid level
Humidity
Light
Pressure
Speed
Gas analysis
Chemical concentration
Acidity and alkalinity
Metallurgical transformation

Many of the classifications appearing in this list are of a gen-

eral nature, and the list would be very much longer if specific classifications were made. For instance, in the matter of gas analysis, the measurement and control of carbon dioxide (CO_2) in flue gases is an important application itself, but the electrical gas analysis apparatus is being used also for measuring and controlling sulphur dioxide (SO_2), oxygen and hydrogen mixtures, nitrogen and hydrogen mixtures, and many others.

Likewise, the chemical concentration recorders and controllers have many widely varying applications. Perhaps the simplest is the condenser leakage recorder which under normal conditions operates on distilled water. When an abnormal condition occurs, such as the breaking of a condenser tube, the instrument records instantly the presence of impurities which may enter the hot well of the condenser because of the leak in the condenser tube. Other applications include recording and controlling the strength of acid baths, such as those used in the mercerizing process.

If all of the applications of the industrial recorders and controllers coming under the classification of acidity and alkalinity were described briefly, it would require a fairly large book. The actual acidity or hydrogen ion concentration of solutions appears now to be almost as important a factor as temperature in process control. Of course, in many processes the importance of hydrogen ion concentration is not as well understood today as the importance of temperature and the apparatus available for recording hydrogen ion concentration and exercising control of this factor are not as well developed or as generally applicable as similar apparatus for temperature recording and control. The future looks very promising. To give you some idea of the processes in which hydrogen ion measurement and control has been found to be important, I will read you the following incomplete list:

Filtration of potable water
Treatment of sewage
Control of leather tanning solutions
Control of the sugar refining process
Treatment of boiler feed water
The baking of bread
The preservation of food stuffs

Manufacture of paper
The electroplating processes
Manufacture of chemicals

The question naturally arises as to why electrical methods are used for recording and controlling these various non-electrical conditions and quantities. Offhand, one might assume that the important reason is the ease with which electrical measurements made at one point can be transmitted to recorders or controllers located at some more convenient point. Undoubtedly, this is a contributing factor, but I do not think it is the outstanding one. Of the factors which I consider more important, it is a little difficult to decide which to put first. These three factors I consider very important, and the order in which I shall read them is without significance as to their relative weight:

First: The electrical method provides means for automatically recording and controlling non-electrical processes, which, without the electrical method could not be automatically recorded or controlled at all.

Second: The best types of electrical recorders are so sensitive that they make possible an accuracy of measurement not obtainable with the more direct, but less sensitive, older methods, and the sensitive elements for the electrical recorders can be made extremely small without great loss of sensitivity.

Third: Some of the measurements formerly considered to be chemical measurements now appear to be fundamentally electrical measurements, so that the electrical method is really the direct, rather than the indirect method.

In explanation of the first point, we might take the example of the chemical concentration recorder as applied to condenser leakage measurements. The non-electrical methods for making such measurements involve chemical titrations of such a nature that it is difficult to conceive how the results could be recorded automatically.

As an example of the second reason, we may take the recording of ordinary temperatures with resistance thermometers. In many instances where temperature conditions are fluctuating more than they should, and a non-electrical instrument is employed, the record appears straight, indicating

steady and satisfactory conditions. An electrical instrument used to measure these same temperature conditions will draw a fluctuating record, indicating the true conditions. The real reason for the difference is, of course, that the non-electrical instrument is not sensitive enough to record many of the temperature changes that can be recorded readily with electrical methods.

Frequently the better sensitivity of the electrical instrument results from the use of a much smaller sensitive element, or detector, than it is possible to use with non-electrical instruments. The small size of the detector minimizes the time lag so that rapid fluctuations can be recorded accurately.

A further advantage of these very small electrical detectors is that they may be put in place without disturbing appreciably the temperature conditions at the point where the temperature is to be measured. For instance, in the temperature processing of canned foods, the temperature detector usually is mounted in one can which is assumed to be representative of a lot. A large temperature detector may disturb the temperature conditions in the specimen can to such an extent that the recorded temperature is substantially different from that in the other cans.

An example of the third reason is the measurement of hydrogen ion concentration. The colorimetric indicators previously used for such measurements are based upon empirical laws. The fundamental condition in which we are interested in such measurements is the concentration of the hydrogen ions, and as we have rather definite information about the electrical charges carried by hydrogen ions, the most direct method for measuring hydrogen ion concentration is that of measuring the electric potential set up by the presence of the hydrogen ions at a hydrogen electrode. Mansfield Clark in his book on "The Determination of Hydrogen Ions" puts it this way:

> "It is the electrical charge which turns an element or group into a virtually new body and at the same time furnishes a handle, as it were, with which we may lay hold on it by electrical devices."

The accepted basic method for measuring hydrogen ion concen-

tration is the electrical method, all color indicators being calibrated against electrical measurements.

You may wish to agree or disagree with my reasons for the trend toward the use of electrical recorders and controllers for the measurement and control of non-electrical quantities and conditions, but whether you agree or disagree does not materially alter the point which I want to make, and that is that there is a distinct trend in that direction.

The Use of Automatic Control

The next trend which I want to discuss is the tendency to equip industrial recorders with automatic control features. Perhaps earlier in my talk, I should have stated the purposes which industrial recorders serve, but I have intentionally delayed discussion of this point until now, in order to emphasize the logic of the tendency to go from automatic recording to automatic control, or a combination of the two in many instances.

Referring now to the purposes of the industrial recorder, these may be classified briefly as follows:

1. To automatically observe and write down as a matter of record the conditions obtaining throughout the course of a process.
2. To give a better guide for regulating a process than can be obtained from indicating instruments.

Referring to the first purpose, this function of the recording instrument is always important as a labor saving device and as a guard against errors, unintentional or intentional, in observing and writing down momentary conditions. Sometimes this purpose of the recorder is of importance because operators cannot be depended upon to observe and record the desired readings as frequently as they should.

The second purpose of the industrial recorder is not so obvious, but is extremely important. In regulating many processes, it is essential for the operator to know not only the condition obtaining at any moment (the function of an indicating instrument), but to know in what direction and to what degree the condition is tending to vary. This additional information is readily obtained from a curve drawn by a

recorder, but is practically impossible to obtain from an indicating instrument. A good operator, in regulating a process from a curve drawn by a recorder, perhaps unwittingly brings into play some highly mathematical considerations. He automatically observes not only the momentary condition and the direction of change of that condition, but he observes also the rate of change of that condition with respect to time (the first derivative) and the rate of change of the rate of change (the second derivative). These observations are very essential to close regulation, particularly in processes involving appreciable time lag.

When precise automatic control of processes involving time lag is attempted, there must be provided in the automatic controller something which is equivalent to this automatic intelligence on the part of a good operator which enables him to observe all of these factors from the record and to adjust the controlled conditions accordingly. And it is just this point that makes the problem of automatic control a difficult, a very interesting, and a scientific one. The objection to manual control guided by a recording instrument is not to be found, I believe, in the inability of an intelligent operator to determine what adjustments are needed at the moment. The objections or limitations to such control, as I see it, are as follows:

First: The high cost of obtaining a sufficient number of intelligent operators to permit each one to be assigned continuously to a particular regulating job.

Second: The inability of any human being to make simultaneously the large number of adjustments that are frequently required to maintain the desired result at all times. Even if each operator had a corps of assistants to help him make the adjustments, it would be practically impossible to relay to these assistants instantaneously and simultaneously the instructions necessary for making the various adjustments; hence the trend toward automatic control in industrial processes.

An intermediate step between automatic recording and automatic controlling is that of equipping industrial recorders

with automatic alarms or signals which make it unnecessary for the operator to watch the record as long as the desired condition obtains within certain limits. Upon the showing of a signal or the sounding of an alarm, or both, the operator goes to the proper recorder, observes the nature and the amount of the correction needed, applies this correction, and then goes about his other duties until another signal or alarm warns him that further adjustment somewhere is needed.

Sometimes automatic controllers are installed without recording instruments, but very frequently the recording feature is retained in a combination recording and controlling instrument. Even with a satisfactory automatic controller in service, something may go wrong with the apparatus or with the process itself, and the record is very useful to show that the automatic controller and everything associated with it in the process are giving the expected results. Frequently program controllers are installed, which automatically carry a process through a rather elaborate cycle of operation. If a record is provided on such a program controller, a glance at the record shows the stage of the cycle at any time.

I think that the foregoing will make it clear that the step from automatic recording to automatic control is a natural and logical one, and although many of you may not have been familiar with the reasons for this trend toward automatic control, I believe that all of you know that this trend has been taking place. Accordingly, I shall not take the time to present proofs that such a trend has been, and is, going on.

It is not my intention to give you a complete discussion of the problem of automatic control in industry, as there would not be sufficient time even if the whole evening were available to devote to this subject alone.

SOME FUNDAMENTAL CONSIDERATIONS IN THE DESIGN OF INDUSTRIAL RECORDERS AND CONTROLLERS.

Now I should like to discuss some of the fundamental considerations in the design of industrial recorders and to follow that with a very brief discussion of some of the fundamental considerations in the design of precision industrial automatic controllers.

Industrial Recorder Design

For the sake of clarity, it will probably be better for me to consider a specific type of measuring circuit in discussing the fundamentals of recorder and controller design. For this purpose, I shall choose the potentiometer circuit, as I have already made reference to this in the early part of my talk. What I have to say, however, will apply equally to other measuring circuits of precision, such as the Wheatstone bridge circuit, the Kelvin bridge circuit, and bridge circuits in general.

The true potentiometer method involves what is known as a null measurement. In a null measurement, the detector stands at its null or zero position when each reading is made and recorded. In other words, a null method is a balance method in which a known quantity is balanced against the unknown quantity and the detector, by standing in its null or zero position, shows that the balance is perfect.

The fundamental correctness of the potentiometer for measuring temperature by means of thermocouples is based primarily on the consideration that it is the voltage produced in a thermocouple that bears a direct relation to the temperature. The current produced in a thermocouple circuit bears a definite relation to the temperature only if the circuit resistance is constant. That part of the circuit comprising the thermocouple itself is subjected to wide variations in temperature, which affect its resistance. As the thermocouple remains in use, it is eaten away, so that its cross-section diminishes and its resistance increases correspondingly. Moreover, the connecting wires between the thermocouple and the recording instrument have an appreciable resistance, which varies with the surrounding temperature, and both the changes in the resistance of the connecting wires and the changes in the resistance of the couple cause changes in the current produced by the thermocouple for any particular temperature. In the null potentiometer, no current flows through the detector at the time when a measurement is made, and it is this fact which makes measurements with the null potentiometer independent of the circuit resistance.

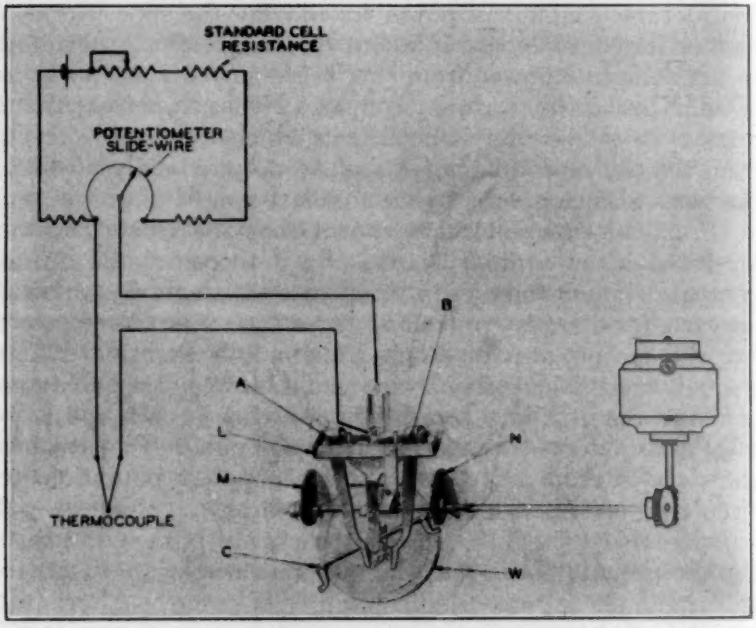
My object in giving you these details of the potentiometer circuit is merely to point out that the ideal method for meas-

uring temperature by means of a thermocouple involves making null measurements, and this fact determines the general design which must be followed in building an industrial thermocouple recorder if it is to operate on this ideal principle. The null recording potentiometer requires a continuous adjustment of an auxiliary source of known voltage, in order to keep the auxiliary voltage at all times equal to the voltage set up in the thermocouple. A satisfactory arrangement for varying the auxiliary voltage requires, among other things, a considerable amount of power for moving the slide wire contact or its equivalent. It is out of the question to attempt to get sufficient power from the feeble currents set up in an ordinary circuit by a thermocouple. Hence, it is clear that a relay type of recorder is needed in which the feeble current from the thermocouple is required to do practically no work, the work all being done by an auxiliary source of power.

Practical experience has shown that the feeble currents produced in an ordinary circuit by a thermocouple do not generate enough force even to operate satisfactorily a pair of contacts for directly controlling the source of auxiliary power. For this reason, and for others which will be mentioned later, a purely mechanical relay arrangement between the galvanometer and the auxiliary source of power has been found to be most satisfactory. I call your attention now to Fig. 1, which shows, in an elementary way, a thermocouple, a potentiometer circuit which provides the auxiliary known voltage, a galvanometer detector, the auxiliary source of power for adjusting the potentiometer and the mechanical relay between the galvanometer and the auxiliary source of power. Alternately at intervals of a second or two, the galvanometer pointer is allowed to swing freely, and then to engage the relay mechanism. When the galvanometer pointer is free, it assumes a position dependent upon the direction and extent of the unbalance in the potentiometer circuit. At this time, it is not allowed to do any work except that minute amount necessary to twist its own suspensions. In the remaining interval of the cycle, the arrangement is such that the auxiliary source of power adjusts the potentiometer setting in the direction and by an amount corresponding to the galvanometer deflection. This arrangement, then, not only has the advantage of requir-

ing practically no work to be done by the galvanometer, not even the closing of an electrical contact, but has the additional very important advantage that the amount of adjustment made by the auxiliary source of power is about proportional to the need for adjustment; in other words, is about proportional to the galvanometer deflection. This proportioning feature permits large adjustments to be made quickly without

FIG. 1.



Recorder mechanism.

danger of over-shooting or hunting in making small adjustments. You will see that what we have here is really an automatic control problem within the recorder itself. This point will be discussed more fully a little later. The manner in which the mechanical relay mechanism operates is as follows: As stated previously, when the galvanometer is free to swing, it deflects through an angle proportional to the degree of unbalance in the potentiometer circuit, and the direction of deflection corresponds to the direction of unbalance. Im-

mediately following the brief interval in which the galvanometer is allowed to deflect, a cam on a shaft driven by the auxiliary source of power raises the lifting bar *L* and pushes the galvanometer pointer up against one or the other of the two bell-crank levers, which are pivoted at *A* and *B*. It will be seen that, for small deflections, the lifting of the bell crank lever is done at a point far from the pivot point, and for maximum deflection of the galvanometer, the lifting is done at a point near to the pivot point. Consequently, for a standard amount of lift, the deflection of the bell crank lever will be proportional to the galvanometer deflection. The deflection of these bell crank levers is transmitted to the clutch bar *C*, so that this clutch bar is deflected an amount corresponding to the galvanometer deflection, and in a direction corresponding to the galvanometer deflection. During this part of the cycle, the clutch bar *C* is disengaged from the clutch wheel *W* by means of another cam, so that the tilting of the bell crank lever of the clutch bar *C* involves only a small amount of power and produces only a small shearing stress on the galvanometer pointer. The clutch cam promptly moves out of the way, allowing the clutch bar *C* to engage with clutch wheel *W*, so that in order to restore the clutch bar *C* to its normal or horizontal position, the clutch wheel, to which is attached the potentiometer slide wire, must be carried with it for a distance corresponding to the deflection of the clutch bar *C* and, hence, corresponding to the deflection of the galvanometer and to the unbalance in the potentiometer circuit. The clutch bar *C* is returned to its horizontal position once each cycle by a large cam driven by the auxiliary source of power. There are two of these large cams *M* and *N*—one for each direction of deflection. The cycle requires one or two seconds, and is repeated continuously, with the result that the galvanometer is maintained in its null or zero position. Hence, whenever the potentiometer is unbalanced, due to a change in temperature of the thermocouple, the balance is promptly reëstablished by moving the slide wire an amount proportional to the unbalance. For all except very large changes in temperature, one adjustment of the slide wire will very nearly reëstablish the balance, and a couple of additional very small steps will reëstablish it exactly. For very large

temperature changes, several full steps may be required, in addition to the final small steps, for obtaining the exact balance.

In this arrangement, the potentiometer contact remains stationary and the slide wire is moved. This is merely a preferred, and not an essential, arrangement. It is obvious that a pen mechanically connected to the moving slide wire will continuously record the position of the slide wire necessary to maintain the balance between the auxiliary source of E.M.F. and the E.M.F. produced by the thermocouple. As each position of the slide wire represents a definite voltage opposed to the voltage of the thermocouple, it is clear that the pen will draw a record of the true thermocouple voltage and, therefore, an accurate record of the temperature.

The auxiliary source of power is a small motor, and this motor is equipped with an electrical speed governor. The governor regulation is good enough to permit using the motor speed as the timing element in the recorder. In other words, the small governed motor drives the chart at the required speed and no clock is needed.

This arrangement, therefore, provides:

First: Means for utilizing a null detector, to gain all of the advantages of a null circuit.

Second: Means for obtaining a record without requiring the feeble currents produced by a thermocouple to do any appreciable work.

Third: Means for proportional control of the auxiliary source of power, so that large adjustments may be made quickly without danger of overshooting or hunting in making small adjustments.

Fourth: Means for driving the chart forward at a constant speed, without the use of a clock.

Fifth: Provision of an excess of power which may be utilized for moving commutators or for operating automatic control contacts or both.

The provision for moving commutators by the recorder mechanism makes possible the multiple-point recorder with which a comparatively large number of temperatures may be recorded on one chart. In the multiple-point recorder,

measurement is made on one thermocouple, and after the temperature is recorded on the chart, and properly identified by a number from a print wheel, a commutator moves to its next position connecting the recorder to another thermocouple, and the process is repeated. Recorders are regularly made with as many as 16 points on the commutator, so that as many as 16 temperatures can be recorded on a single chart. There are many variations in the design of recorders employing commutators for switching purposes, but I shall not take the time to refer to these, as I want to use what time is still available to tell you something about some of the fundamentals of the design of automatic controllers. Before taking this up, however, there is one point which I should mention in connection with the recorder circuits. I purposely avoided mention of it up until now.

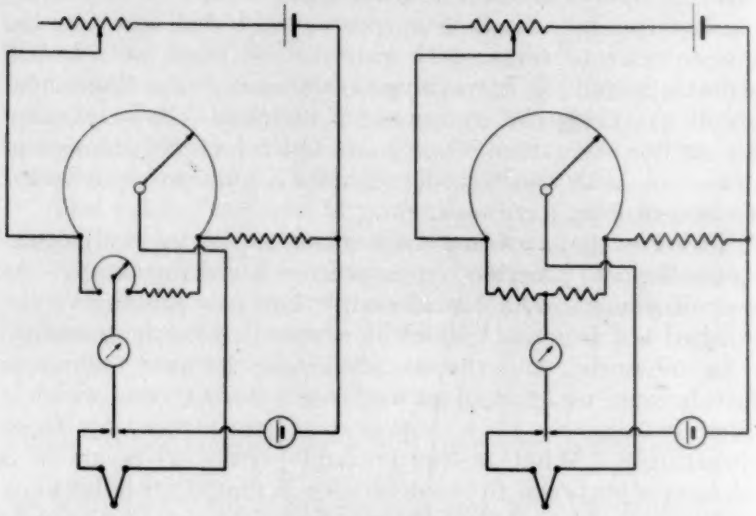
I refer to the provision of automatic means for compensating for the cold junction temperature in a thermocouple. As most of you know, a thermocouple has two junctions—the so-called hot junction[1] which is exposed to the temperature to be measured, and the so-called cold junction,[2] which is there because we know of no way to get rid of it, and which is normally exposed to a temperature approximating room temperature. What a thermocouple really gives us is a voltage proportional to the difference in temperature between the hot junction and the cold junction. Therefore, the temperature of the hot junction is not made known by a measurement of the thermocouple voltage unless we know the temperature of the cold junction. One way to take care of the cold junction temperature is to place the cold junction in a constant temperature chamber of some sort, but a more convenient way, when working with the potentiometer, is to equip the potentiometer circuit with an automatic cold junction compensator. The way in which this is done is shown in Fig. 2. At the left is shown a split circuit potentiometer with two adjustable contacts. The upper slide wire is calibrated on the assumption that there is zero voltage produced at the cold junction, in which case the position of the contact slider is a measure of the temperature of the hot junction. Actually, a

[1] Preferred name measuring junction.

[2] Preferred name reference junction.

voltage is always produced at the cold junction, but this is made equivalent to zero by setting the lower sliding contact at a point representative of the temperature of the cold junction. This particular arrangement, however, would involve frequent observation of the cold junction temperature and frequent manual adjustment of the cold junction contact. Such

FIG. 2.



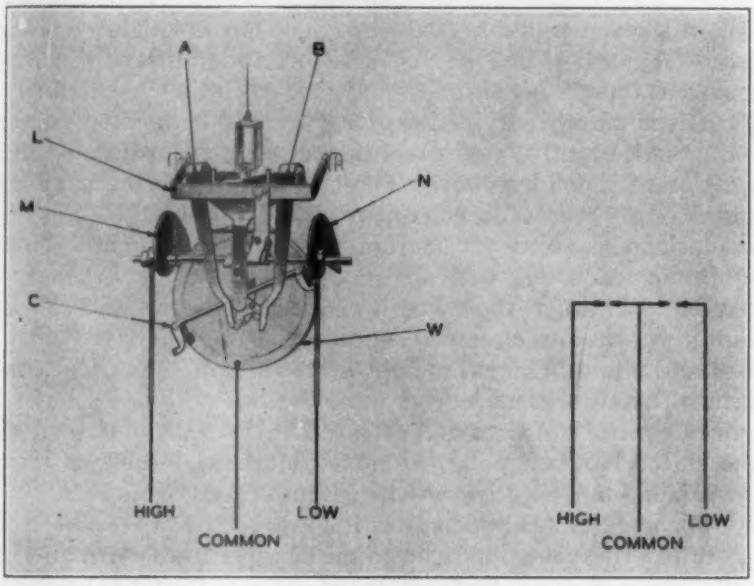
Thermocouple cold-junction compensation.

operations are eliminated, and the cold junction compensation is made fully automatic by the use of the circuit shown at the right. Here the cold junction contact is fixed instead of movable, but the relative amount of resistance to the left and to the right of the fixed contact is made variable by making the left hand resistance coil of material having a high temperature coefficient of resistance. This coil is really a resistance thermometer, and it is placed in close proximity to the cold junction so that it and the cold junction will always be at the same temperature. Then, regardless of what the cold junction temperature is, the potentiometer automatically measures the temperature at the hot junction.

Precision Controller Design

In my discussion of recorder mechanisms, I mentioned two things which have a direct bearing on the problem of automatic control. One was that, in the particular recorder mechanism described, there is involved an automatic control problem, that is, the problem of controlling an auxiliary source of electromotive force of known value, in order to

FIG. 3.



Controller mechanism.

maintain a balance between the known electromotive force and the unknown electromotive force of the thermocouple. The other was that the type of control used for this purpose involved a proportioning feature, which permits making large adjustments quickly without the danger of overshooting when making small adjustments. I now wish to show how the parts of this recorder mechanism, which are used for obtaining the proportional control within the recorder, may be used for obtaining proportional control in general. Fig. 3 illustrate as modified recorder mechanism designed for automatic control

purposes. The galvanometer, and its associated lifting bar and bell crank levers, are exactly the same as in the recorder mechanism. The galvanometer circuit is completed through whatever measuring circuit is necessary to measure the direction and extent of departure from the normal state of the condition to be controlled. In this modified arrangement, the clutch wheel *W* is held stationary by a clamp, and the common lead of a 3-wire control circuit is connected to the clutch wheel. The face of the clutch wheel, which is slightly roughened in the recorder mechanism, is made smooth and serves as a slip ring in the controller mechanism. The two cork clutch shoes carried by the clutch bar *C* in the recorder mechanism are replaced by sliding brushes in the controller mechanism. Thus, the clutch bar is always electrically connected to the common lead of a 3-wire control circuit, regardless of the position of the clutch bar. The two large restoring cams *M* and *N* of the recorder mechanism are modified to the extent of insulating them from the shaft on which they are mounted and equipping each one with an independent sliding brush connection. Through these brush connections, one main cam is kept in continuous electrical contact with one selective control lead and the other cam is kept in continuous electrical contact with the other selective control lead. Thus we have a pair of control contacts as illustrated at the right of the figure, the clutch bar being the common contact and the two large cams being the selective or "high" and "low" contacts.

The cycle of operation is as follows:

When the controlled condition departs from normal, the galvanometer will deflect. The direction of the galvanometer deflection will indicate the direction of the departure, and the angular displacement of the galvanometer will be a measure of the extent of the departure. As pointed out previously in discussing the recorder mechanism, the bell crank levers and the cams operate to deflect the clutch bar through an angle proportional to the angular displacement of the galvanometer. Hence, a large deflection of the clutch bar indicates a large departure from normal and a small deflection of the clutch bar indicates a small departure from normal. Also the direction of deflection of the clutch bar corresponds to the direction of the departure. After the clutch bar has been

deflected, one or the other of the large cams *M* and *N* (depending upon the direction of departure) will make contact with the clutch bar, and this contact will be maintained until the clutch bar has been returned to its normal or horizontal position. If the deflection of the clutch bar has been large, contact will be made well around toward the heel of the cam, so that the contact will be of rather long duration. If the deflection of the clutch bar has been small, the contact will not be made until some point up near the toe of the cam is reached; hence, the duration of the contact will be very brief. For intermediate deflections, the duration of contact will be in proportion. Usually these contacts operate, through relays, a control motor, with the result that for small departures from normal, the motor will make only a few revolutions during each control cycle, whereas for large departures, the control motor will make many revolutions during each control cycle, the number of revolutions being substantially proportional to the extent of the departure.

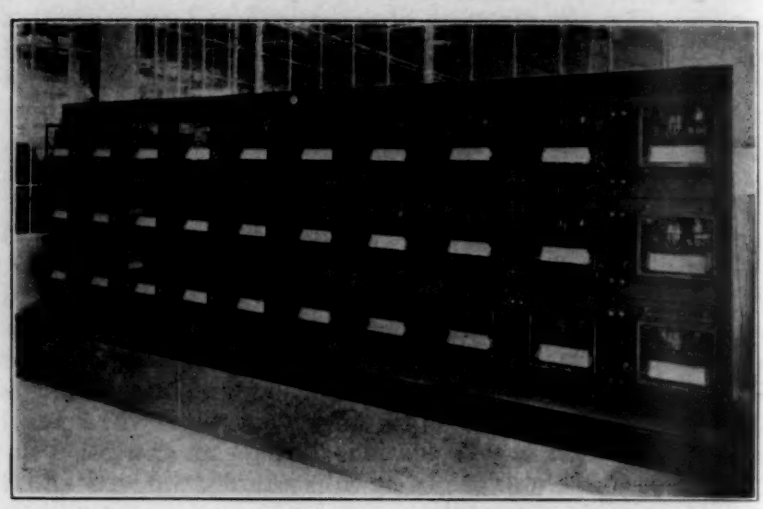
This proportional action of the controller is one of the fundamental principles of correct automatic control of any condition or quantity in which time lag is involved, and usually all control problems involve time lag in some degree.

I do not mean to imply that this proportional action of the controller is a "cure-all" for the various obstacles encountered in automatic control problems. It is only one of the fundamental principles on which correct automatic control is based. However, it is perhaps the outstanding fundamental principle.

Various other types of control systems regularly are actuated by this recorder mechanism, some of these being more simple and some more complex than the one I have described. It would require a full evening to discuss various types of controllers, and the almost infinite variety of control problems for which they are used, so I shall not attempt it now.

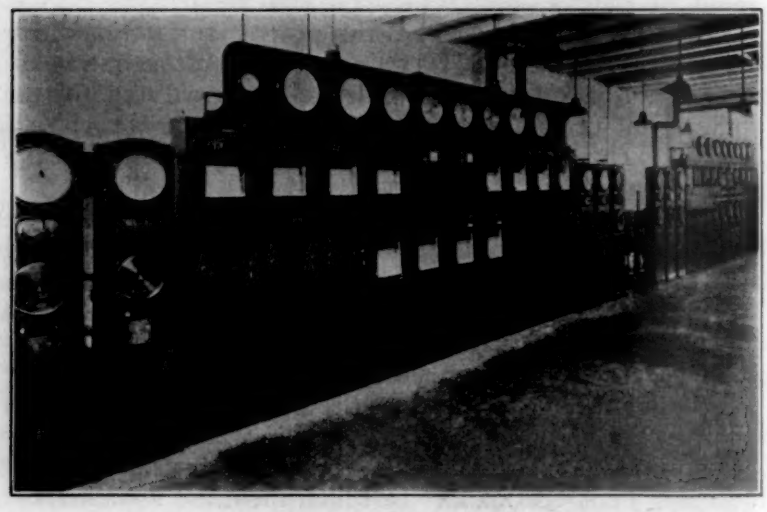
I think a fairly good index of the appreciation of a piece of apparatus by its owner is the manner in which he takes care of that piece of apparatus. In considering industrial recorders and controllers, the manner in which these are installed and maintained by the users gives, I believe, some indication of the respect which the purchaser has for these instruments.

FIG. 4.



Recorder installation in heat-treating department of large automobile plant.

FIG. 5.



Recorder installation on cracking stills in oil industry.

FIG. 6.

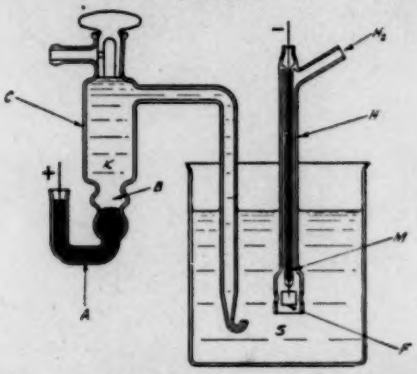
Recorder installation in automobile plant.

FIG. 7.

Recorder installation in heat treating room of large automobile plant.

Therefore, I want to show you a few slides illustrating installations of industrial potentiometer recorders and controllers.

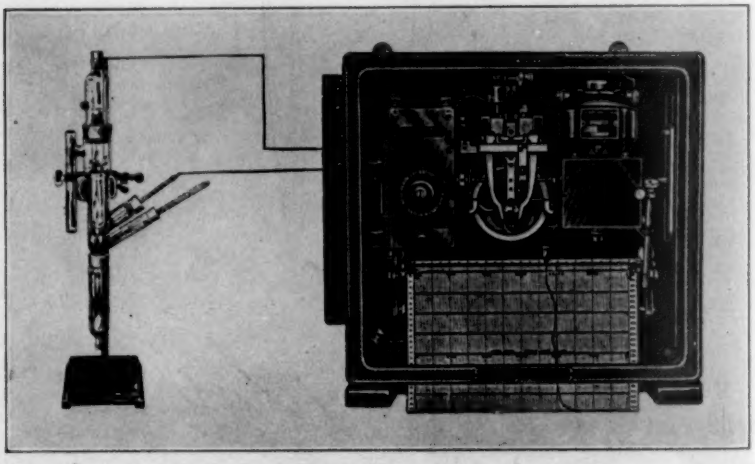
FIG. 8.



Schematic arrangement of electrodes for electrical measurement of hydrogen ion concentration.

You may be interested in knowing just how electrical recorders and controllers are applied to the measurement and control of non-electrical conditions. I have prepared a few

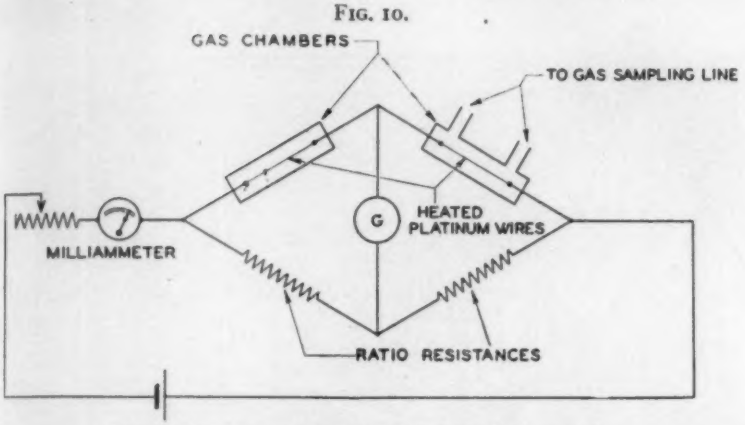
FIG. 9.



Hydrogen ion recorder and electrodes.

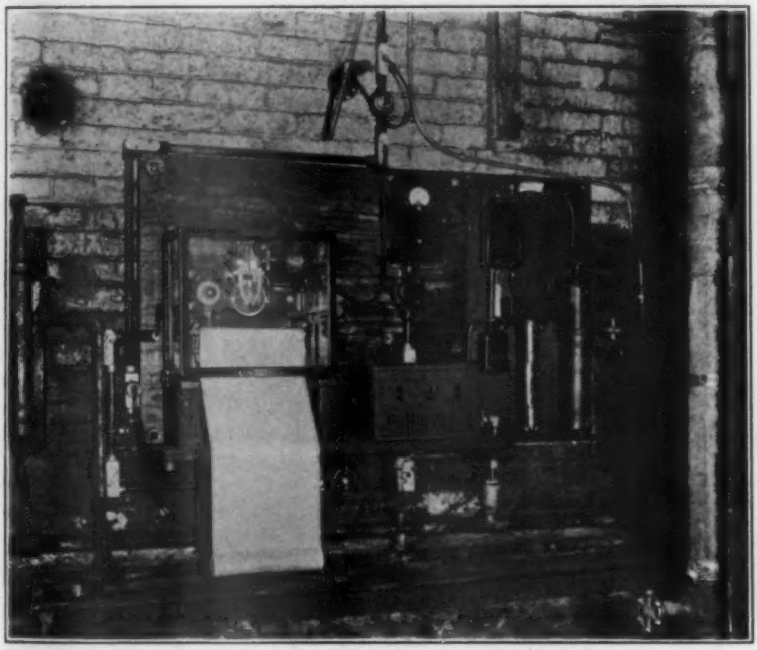
slides showing how the translation is made from the non-electrical condition to the electrical measurement in some applications.

FIG. 10.



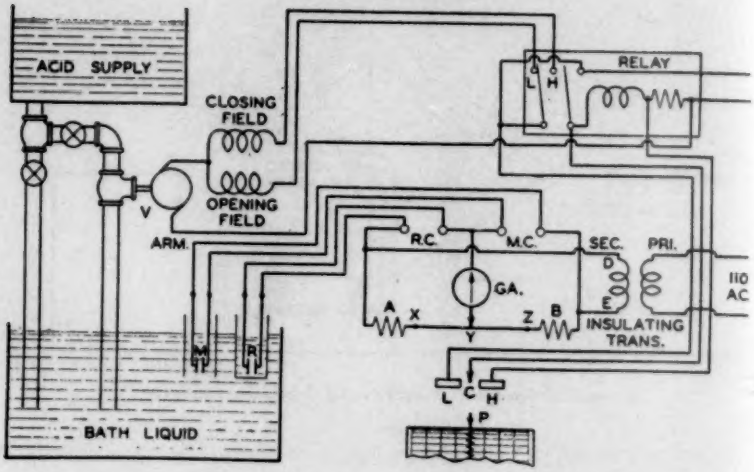
Schematic arrangement of apparatus for electrical gas analysis.

FIG. 11.



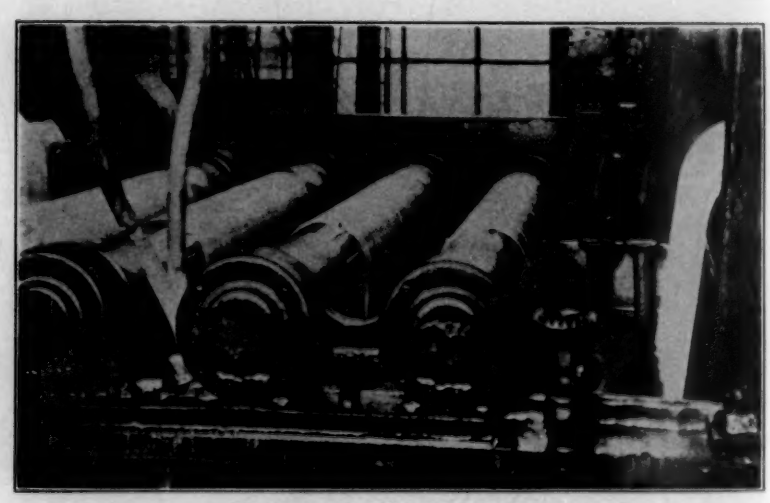
Installation of electrical recorder for CO_2 measurements.

FIG. 12.



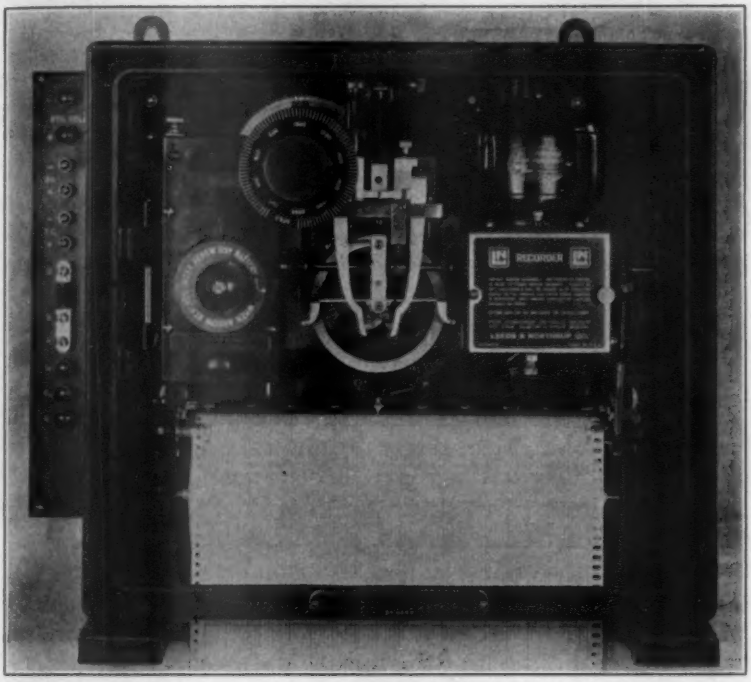
Schematic arrangement of apparatus for electrical measurement of chemical concentration.

FIG. 13.



Installation photograph showing electrodes in mercerizing tank or controlling acid concentration.

FIG. 14.



Leeds and Northrup recorder-controller.

Reliability of Industrial Recorders and Controllers

Perhaps in listening to the descriptions of these recorder and controller mechanisms, you have gained the impression that they are very complex. Possibly you may have gone further in your thinking and concluded that, because they are complex, they are likely to be unreliable. Therefore, in concluding this talk, I should like to say a few words about the reliability of mechanisms such as these. To help make the matter clear, I should like to use the expression "factor of reliability," which I shall define as follows: If a device works perfectly, that is, if it has no failures at all, the factor of reliability would be 1; if the device failed to function 10 per cent. of the time, the factor of reliability would be .9. Of course, it is practically impossible to assign a specific figure to each element of a recorder mechanism which can be considered

as its "factor of reliability." However, each element undoubtedly does have a fairly definite factor of reliability, even though these factors may be difficult to determine in an accurate and practical way. Assuming, for the moment, that factors of reliability can be assigned to each element of a mechanism, let us take a mechanism having in it only two elements, each with a factor of reliability of .5. The net factor of reliability of the assembly should be the product of the two or .25. Let us now take an assembly having in it five elements, each with a factor of reliability of .9. The net factor of reliability should be .9 multiplied by itself four times, which is approximately .6. Hence, we have a very simple assembly with a net factor of reliability of only .25, and a much more complex assembly having over twice the number of parts, with a net factor of reliability of .6, which is more than twice as good as the factor of reliability for the simpler assembly. These calculations do not take into account the law of probability. They assume the worst possible combination of failures. In justification of this, I may say that it is the usual practice in designing measuring apparatus to ignore the law of probability and to assume that all errors and failures will combine in the worst possible way. All of this discussion about reliability is very simple and obvious and hardly worth calling to your attention, except for the fact that simplicity and reliability are often considered to be inseparable conditions, whereas, in fact, there is no real ground for connecting the two. Stated another way, it is always wrong to substitute a single element of doubtful reliability for a combination of two or more elements, each having a high degree of reliability.

While this reasoning about reliability of complex mechanisms may be acceptable to scientists and engineers, we cannot be content with this reasoning as applied to industrial mechanisms, unless the reasoning is acceptable also to the practical industrialist. A natural question is whether practical minded men in industry will accept fairly complex mechanisms even though they do operate on fundamentally correct principles, and even though they are so designed as to have higher factors of reliability than can be obtained in simpler mechanisms. It is gratifying to be able to say that industry is not only accepting such mechanisms, but is accepting them willingly—I think I may say eagerly.

NOTE ON THE DERIVATION OF THE EQUATION OF STATE OF A MIXTURE OF SUBSTANCES FROM THE EQUATIONS OF STATE OF THE CONSTITUENTS.

BY

R. D. KLEEMAN, B.A., D.Sc.

IN a previous paper [1] the writer has given a method of determining the equation of state of a mixture of substances from the equations of state of the constituents. The functions of the absolute temperature T which the equations contain should be looked upon, it was stated (p. 512), as entirely independent of each other. The importance of this statement was not completely realized when the paper was written, and it will therefore be followed up in this paper, and the subject otherwise further developed.

Let the equation of state of a pure substance a be written in the form

$$p_a = \phi(T) \cdot \psi\left(\frac{A_1 M_a}{v}, \frac{A_2 M_a}{v}, \cdots\right), \qquad (1)$$

where p_a, v, and M_a, denote the pressure, volume, and mass of the substance respectively, and A_1, A_2, $\cdots$ functions of T. The form of the function ψ is such that it becomes zero when $M_a = 0$, since under these conditions $p_a = 0$ as should be the case. Next let us suppose that the mass of the substance a is divided into the two masses M_a' and M_a'', in which case the equation becomes

$$p_a = \phi(T) \cdot \times \psi\left(\frac{A_1 M_a' + A_1 M_a''}{v}, \frac{A_2 M_a' + A_2 M_a''}{v}, \cdots\right). \qquad (2)$$

Let us now suppose that the molecules of mass M_a'' become molecules b, whose mass will now be written M_b, and M_a that of the remaining molecules a. Without enquiring what

[1] JOUR. FRANK. INST., 206 (4), 511 (1928).

change this should introduce into the form of the equation, it follows immediately that when $M_b = 0$ the equation should assume the form (1). From this it follows that the function ψ will involve only terms in which M_a or M_b, or both, occur as factors. Hence when the molecules a belonging to the mass M_a'' in equation (2) become molecules b the factors of M_a'', or M_b, undergo an appropriate change so that the experimental facts relating to the mixture are represented. Now this is in general possible only if these factors may vary in form entirely independently of each other. Suppose that this is the case, and let us write these factors $B_1, B_2, \cdots$, in which case equation (2) assumes the form

$$p = \phi(T) \cdot \times \psi\left(\frac{A_1M_a + B_1M_b}{v}, \frac{A_2M_a + B_2M_b}{v}, \cdots\right), \quad (3)$$

where p denotes the pressure under the new conditions, i.e. the pressure of the mixture. If we write $M_a = 0$ the equation becomes

$$p_b = \phi(T) \cdot \psi\left(\frac{B_1M_b}{v}, \frac{B_2M_b}{v}, \cdots\right) \quad (4)$$

and is the equation of state of the substance b, in which the temperature functions $B_1, B_2, \cdots$, are independent of each other. The functions $B_1, B_2, \cdots$ in equation (3) do not involve constants depending on the nature of the substance a. For these constants would not disappear when writing $M_a = 0$, or they could not be equated to zero, since they are absolute constants. The equation therefore does not contain functions of T involving the nature of both the substances a and b. It follows now from considerations of symmetry that the functions $A_1, A_2, \cdots$, are also independent of each other.

It can easily be shown now that the function $\phi(T)$ is independent of the nature of substances. For suppose that it contains constants depending on the nature of the substance a and b. When $M_b = 0$ the function should be independent of the substance b. This cannot be realized by writing the constants referring to the substance b equal to zero, for the values of these constants cannot be of a variable nature. Hence the function contains only absolute constants.

The equation of state of the pure substance c is

$$p_c = \phi(T)\cdot\psi\left(\frac{C_1M_c}{v}, \frac{C_2M_c}{v}, \cdots\right), \tag{5}$$

where C_1, C_2, $\cdots$, are functions of T independent of each other. The equation of state of a mixture of substances a, b, c, is therefore

$$p = \phi(T)\cdot\psi\left(\frac{A_1M_a + B_1M_b + C_1M_c}{v}, \frac{A_2M_a + B_2M_b + C_2M_c}{v}, \cdots\right) \tag{6}$$

and so on.

It will appear now that the equation of state of a mixture can be deduced from the equations of state of the constituents only if the latter equations are written in a form so that the functions of the temperature they involve are independent of each other. It does not follow however from this that no relations may exist between these functions. If any exist they should be expressed by separate equations. For an equation of state should evidently be initially treated in the most general manner possible, or independently of any relations that may exist between its coefficients, and if any relations exist these should be expressed by separate equations.

The relations that arise between the coefficients of an equation of state on transforming it algebraically by expansion, or otherwise, do not come under this heading. For the new coefficients that arise in this manner cannot be treated *a priori* as independent. The equation of state must therefore be used in its simplest algebraical form, or the form in which the coefficients representing functions of the temperature are *algebraically independent* of each other.

An example will help to make the matter clear. If the equations of state of the substances a and b are

$$p_a = A_1\frac{M_a}{v}\Big/\left(1 + A_2\frac{M_a}{v}\right) + A_3\left(\frac{M_a}{v}\right)^2, \tag{7}$$

$$p_b = B_1\frac{M_b}{v}\Big/\left(1 + B_2\frac{M_b}{v}\right) + B_3\left(\frac{M_b}{v}\right)^2, \tag{8}$$

where A_1, A_2, A_3, B_1, B_2, B_3 are functions of T independent of each other, the equation of state of the mixture will be

$$p = \left(A_1\frac{M_a}{v} + B_1\frac{M_b}{v}\right)\Big/\left(1 + A_2\frac{M_a}{v} + B_2\frac{M_b}{v}\right) + A_3\left(\frac{M_a}{v}\right)^2 + \sqrt{A_3B_3}\frac{M_aM_b}{v^2} + B_3\left(\frac{M_b}{v}\right)^2 \quad (9)$$

according to what has gone before.

Next suppose that each side of equations (7) and (8) is expanded in powers of M_a/v and M_b/v. The coefficients of these powers will evidently not be independent of each other. Hence we cannot combine these coefficients in the same way as was just done with the coefficients of the original equations.

An important corollary follows from the foregoing results. If we assume a certain form for the equation of state of a pure substance and find that the equation of state of a mixture deduced from these equations does not agree with the facts, it is an indication that the form assumed is not correct.

This point is of interest in view of some numerical calculations in connection with the equations of state of mixtures deduced by different methods, which have been carried out by Deming and Shupe.[2] When employing the author's method they assumed that the equation of state of a substance consists of a series of powers of M/v containing four terms. The corresponding equation of state of a mixture deduced for two constituents did not give so good an agreement with the facts as another equation deduced in a different way. In the light of the foregoing investigation it appears that therefore the equation of state strictly cannot have this form, as we might expect for many other reasons.

The proof that the equation of state has the form indicated by equations (3) and (6) may be elaborated, and we shall therefore now give the matter further consideration. It can be shown that equation (3) cannot contain any functions other than A_1, A_2, $\cdots$ B_1, B_2, $\cdots$, or no functions which depend on both the substances a and b. For let ϕ_1 $(T, a_1, a_2, \cdots b_1, b_2, \cdots)$ be such a function, where a_1,

[2] JOUR. FRANK. INST., 208 (3), 389 (1929).

$a_2, \cdots b_1, b_2, \cdots$ are constants referring to the nature of the substances a and b respectively. Now suppose that the substance b becomes the same as the substance a, in which case the function becomes $\phi_1 (T, a_1, a_2, \cdots a_1, a_2, \cdots)$. Equation (3) now refers to the substance a, becoming in fact equation (2); it contains the foregoing function, which is therefore one of the functions $A_1, A_2, \cdots$. Next suppose that the substance a becomes the same as the substance b, in which case the function becomes $\phi_1 (T, b_1, b_2, \cdots b_1, b_2, \cdots)$. Equation (3) now becomes of the form (4); it contains the foregoing function, which is therefore one of the functions $B_1, B_2, \cdots$. Thus a function of T involving the nature of both constituents a and b cannot occur in equation (3) unless one of the functions $A_1, A_2, \cdots$ has the same functional form in respect to T as one of the functions $B_1, B_2, \cdots$. Hence if this is not the case for any pair of these quantities, the equation of state of the mixture has the form given by equation (3).

It will now be shown that this is true whatever the relation between the foregoing functions. Suppose that the substance b is removed from the mixture. This does not mean that each of the constants $b_1, b_2, \cdots$ now becomes zero in the function $\phi_1 (T, a_1, a_2, \cdots b_1, b_2, \cdots)$, since they are invariants in nature. The function must therefore consist of two parts $\phi_2 (T, a_1, a_2, \cdots)$ and $\phi_3 (T, b_1, b_2, \cdots)$, the former involving the constants $a_1, a_2, \cdots$ and the latter the constants $b_1, b_2, \cdots$, and the former function must vanish from the equation when $M_a = 0$, and the latter when $M_b = 0$. This is realized only if the former function has M_a as a factor, and the latter function M_b as a factor. Thus the function $\phi_1(T, a_1, a_2, \cdots)$ corresponds to one of the functions $A_1, A_2, \cdots$, and the function $\phi_3(T, b_1, b_2, \cdots)$ to one of the functions $B_1, B_2, \cdots$. It follows therefore that equation (3) does not contain any functions of T which do not occur in the equations (1) and (4). This may also be proved for equation (6), and for an equation involving any number of constituents, in connection with the equations of state of constituents.

The objections to the form of equation (3) expressed by Deming and Shupe in their paper mentioned (pp. 389–390) should be met by the foregoing investigation.

The following method of dealing with this point may be noticed. Let a mass M_c of the substance c be added to the pure substance a. The corresponding equation of state may be written

$$p = \phi(T) \cdot \psi\left(\left(\frac{M_a + M_c}{v}\right)A_1', \left(\frac{M_a + M_c}{v}\right)A_2', \cdots\right)$$

where A_1', A_2', $\cdots$ are functions of T, M_a, M_c and constants depending on the nature of the substances a and c. The functional nature of each of these functions in respect to T is likely to differ from the functional nature of each of the functions B_1, B_2, $\cdots$. It might be possible to find a substance c which will ensure that this is the case. Hence if this mixture is added to the substance b, we have from what has gone before for the equation of state

$$p = \phi(T) \cdot \psi\left(\frac{A_1'(M_a + M_c) + B_1 M_b}{v}, \frac{A_2'(M_a + M_c) + B_2 M_b}{v}, \cdots\right). \quad (10)$$

On obtaining the limiting form of this equation corresponding to M_c becoming equal to zero, we obtain $A_1' = A_1$, $A_2' = A_2$, $\cdots$ and the equations reduces to equation (3).

In general therefore as far as the foregoing investigation is concerned the functions A_1, A_2, $\cdots$ are independent of the functions B_1, B_2, $\cdots$, and so on. It is however very likely from considerations of symmetry that the functions A_1, B_1, C_1, $\cdots$ have the same functional form in respect to T, and that the functions A_2, B_2, C_2, $\cdots$ have the same functional form in respect to T, and so on. It does not seem that this can be proved in a general way at present, and the relation may therefore be introduced only as an assumption.

It is also not unlikely that the functions A_1, B_1, C_1, $\cdots$ may have in general the forms a_1X_1, b_1X_1, c_1X_1, $\cdots$ where X_1 is a function of T and absolute constants, and a_1, b_1, c_1, $\cdots$ are constants depending on the nature of the substances a, b, c, $\cdots$ respectively, and that the functions A_2, B_2, C_2, $\cdots$ have in general the forms a_2X_2, b_2X_2, c_2X_2, $\cdots$ where X_2 and a_2, b_2, c_2, $\cdots$ have meanings similar to the quantities X_1

and $a_1, b_1, c_1, \cdots$ and so on. This relation is a special case of the foregoing. It cannot therefore be proved to hold generally, and therefore when used involves an assumption.

It thus appears that the equation of state (6) of a mixture may be given two special forms of interest, which are however of the nature of assumptions. It appears further upon investigation that the second of the special forms has been used previously but in a somewhat restricted manner, and is usually cited as the method of linear combination of parameters.[3] The parameters in the foregoing considerations are represented by $a_1, b_1, \cdots a_2, b_2, \cdots$, etc. Deming and Shupe in their paper use in this connection the particular form of the equation of state of a substance proposed by Beattie and Bridgeman. They also make the statement that the method of linear combination of parameters rests on a firmer theoretical basis than the method proposed by the writer. It will appear from the investigation in this paper that just the opposite is true. It may of course be proved some time in a general way that the coefficients $A_1, B_1, \cdots A_2, B_2, \cdots$, etc. in equations (1), (4), (5), and (6) may be given the forms described, at present such a proof is not available. The proof when given is likely to involve the general kinetic properties of matter and the existence of the molecular forces of interaction. It should be remarked here that if the facts can be well represented by such a modified equation this would not be reliable evidence that its form is correct, for reasons that will be obvious.

We have seen that the form of equation (1) should be such that the functions $A_1, A_2, \cdots$ are algebraically independent of each other. Possibly this may be realized only if the equation is given the form

$$\Phi\left(p_a, \frac{A_1 M_a}{v}, \frac{A_2 M_a}{v}, \cdots\right) = 0. \qquad (11)$$

The equation of state of a mixture corresponding to equation (3) is accordingly

[3] See James A. Beattie, *J. Am. Chem. Soc.*, 51, 20 (1929); Deming and Shupe, J. FRANK. INST., 208 (3), 390 (1929). Compare equations (7), (8), and (9) in this connection.

$$\Phi\left(p, \frac{A_1M_a + B_1M_b}{v}, \frac{A_2M_a + B_2M_b}{v}, \cdots\right) = 0. \quad (12)$$

On solving these equations in respect to p_a or p we may obtain several roots for these quantities. They would correspond to isomers of the pure substance and the mixture. Some of these roots may be imaginary for certain values of T, M_a, M_b, and v. These remarks apply of course in general.

It may be said, I think, that the foregoing investigation and that given in a previous paper furnishes the fundamental theory of the deduction of the equation of state of a mixture from the equations of state of the constituents. The fundamentals, or axioms, involved are, (1) when the masses of a mixture of substances are zero the pressure is zero, (2) the pressure is independent of the total mass, (3) when the masses of some of the constituents are zero the functional forms of ϕ and ψ in equation (6), or of Φ in equation (12), are not changed, and hence when all the masses of the constituents are zero except one the equation becomes the equation of state of the latter constituent in the pure state. (4) The temperature functions involved are also not changed under these conditions.

The equation of state (6) deduced in this paper is of a somewhat more general character than that deduced before by the writer, on account of the introduction of the functional factor $\phi(T)$. The various deductions that were made from the equation previously given, and its various applications, one of which is the determination of the physical and chemical properties of a mixture of substances, or of a compound, by means of the equations of state of the constituents, hold also for this equation. It also fits in, similarly as the equation previously given, with the equation of state of a mixture deduced thermodynamically [4] to hold at the absolute zero of temperature.

SCHENECTADY, N. Y.

[4] JOUR. FRANK. INST., 206 (5), 631 (1928).

DYNAMICAL ANALYSIS OF MACHINES.*

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ELASTIC REACTIONS IN TERMS OF VELOCITY RATIOS.

A DIRECT derivation of the elastic constant of a generalized spring for a system with one degree of freedom can be obtained in terms of the velocity ratios directly.

$$\Phi_{s\theta} = -\frac{\partial V}{\partial \theta} \qquad \text{where} \qquad V = \tfrac{1}{2}c(S_0 + \Delta S)^2,$$

S being the displacement of the spring element from its equilibrium value S_0. Therefore,

$$\Phi_{s\theta} = -C(S_0 + \Delta S)\frac{\partial S}{\partial \theta} = -CK_{s\theta}(S_0 + \Delta S).$$

Now the velocity ratio itself is a function of θ, so that if K_{s_0} is the equilibrium value, then:

$$K_{s\theta} = K_{s\theta_0} + \left(\frac{\partial K_{s\theta}}{\partial \theta}\right)_0 \Delta\theta, \qquad \text{where} \qquad \Delta\theta = \theta - \theta_0;$$

$$\therefore \quad \Phi_{s\theta} = -CS_0K_{s\theta_0} - C\left[K_{s\theta_0}\Delta S + S_0\left(\frac{dK_{s\theta}}{d\theta}\right)_0 \Delta\theta\right]$$

$$= -\Phi_0 - C\left[K_{s\theta}^2 + S_0\left(\frac{dK_{s\theta}}{d\theta}\right)_0\right]\Delta\theta.$$

The dynamic equation of oscillation is:

$$A\ddot{\theta} + \frac{1}{2}\frac{dA}{d\theta}\dot{\theta}^2 = P - \Phi_0 - C\left[K_{s\theta}^2 + S_0\left(\frac{dK_{s\theta}}{d\theta}\right)_0\right]\Delta\theta,$$

* Extension of a portion of a dissertation for the degree of doctor of philosophy submitted to Clark University, 1928.

Continued from p. 36, vol. 209, January, 1930.

where the equilibrium value of P is $P_0 = \Phi_0$, so that, for small oscillation about the equilibrium value,

$$A\frac{d^2\theta}{dt^2} + C\left[K_{s\theta}^2 + S_0\left(\frac{dK_{s\theta}}{d\theta}\right)_0^1\right]\Delta\theta = \Delta P,$$

where ΔP is some harmonic function.

Evidently the equivalent spring constant is:

$$C_e = C\left[K_{s\theta}^2 + S_0\left(\frac{dK_{s\theta}}{d\theta}\right)_0\right].$$

MACHINE VIBRATING SYSTEM WITH ONE DEGREE OF FREEDOM.

We will now consider the oscillation of a mechanical system as shown in Fig. 3.

It is evident due to the hanging weight mg the system will be static equilibrium at some crank angle θ_0. We wish to consider the nature of the oscillations about this equilibrium position.

The inertia of the system is A, where

$$A = I + mR^2 + M\left(\frac{\partial y}{\partial\theta}\right)^2,$$

since

$$y = r'(1 - \cos\theta), \qquad \frac{\partial y}{\partial\theta} = r'\sin\theta;$$

$$\therefore\ A = I + mR^2 + Mr'^2\sin^2\theta.$$

If T is the torque due to the spring reduced to the coördinate θ, then:

$$T = Cx\frac{\partial x}{\partial\theta}, \qquad x = r(1 - \cos\theta),$$

so that the dynamic equation of the system is:

$$mgR - T = A\ddot{\theta} + \frac{1}{2}\frac{dA}{d\theta}\dot{\theta}^2,$$

but for the equilibrium position:

$$T_0 = mgR = Cx_0\left(\frac{\partial x}{\partial\theta}\right)_0$$

and since we may write T for any displacement around the position, by

$$T = T_0 + \frac{\partial T}{\partial \theta} \cdot \Delta\theta,$$

we have finally,

$$-\frac{\partial T}{\partial \theta}\Delta\theta = A\frac{d^2\Delta\theta}{dt^2},$$

since the square term may be omitted for small oscillations.

The equation of oscillation is, therefore,

$$(I + mR^2 + Mr'^2 \sin^2\theta_0)\frac{d^2\Delta\theta}{dt^2} + Cr^2(\cos\theta_0 - \cos^2\theta_0 + \sin^2\theta_0)\Delta\theta = 0.$$

In terms of velocity ratios,

$$\sum mK_0^2\ddot{\theta} = -c\left[K_{x\theta_0} + x_0\left(\frac{dK_{x\theta}}{d\theta}\right)_0\right]\Delta\theta;$$

now,

$$\sum mK_0^2 = I + mR^2 + MK_{y\theta_0}^2 = I + mR^2 + Mr'^2\sin^2\theta_0$$

$$x_0 = r(1 - \cos\theta_0), \qquad K_{x\theta_0} = r\sin\theta_0,$$

$$\left(\frac{dK_{x\theta}}{d\theta}\right)_0 = r\cos\theta_0, \qquad \ddot{\theta} = \frac{d^2\Delta\theta}{dt^2}.$$

$$\therefore\ (I + mR^2 + Mr'^2\sin^2\theta_0)\frac{d^2\Delta\theta}{dt^2} = -cr^2(\sin^2\theta_0 + \cos\theta_0 - \cos^2\theta_0)\Delta\theta,$$

where

$$mgR = cx_0K_{x\theta_0}, \qquad \therefore\ cr^2(1 - \cos\theta_0)\sin\theta_0 = mgR.$$

PRINCIPLES OF LINEAR AND ANGULAR MOMENTUM IN TERMS OF VELOCITY RATIOS AND THEIR DERIVATIVES.

In order to determine the external or overall reactions in a mechanism with one degree of freedom, we may use the principles of linear and angular momentum for the assemblage of members at any given configuration. It is particularly convenient to express the equations of motion for the system

in terms of the velocity ratios and their derivatives with respect to the fundamental coördinate.

In considering the external or overall reactions, we include all reactions of constraints other than internal reactions between the moving parts. In other words, all external constraints, such as bearing and frame reactions, etc., on the moving mechanism are to be included with the external forces when using the equations of linear and angular momentum of the system as a whole. It is particularly important to differentiate between the system of forces of the previous generalized equations of motion and the system of forces to be used in the equation of linear and angular momentum. In the former, we include all forces which do work, whether external or internal, so that the reactions of the constraints, whether internal or external, disappear. In the latter, we include only the external forces acting on the moving parts, but these include reactions of constraint of an external character as well, while all internal reactions disappear in the equations of motion.

Let x and y be the coördinates of the c. of g. of any member with respect to any given origin, usually an axis of a shaft, m the mass of the member and I its moment of inertia about its c. of g. Then,

$$\Sigma X = \frac{d}{dt}(\Sigma m\dot{x}), \qquad \Sigma \bar{Y} = \frac{d}{dt}(\Sigma m\dot{y}),$$

$$\Sigma L = \frac{d}{dt}\Sigma m(x\dot{y} - y\dot{x}) + \frac{d}{dt}\Sigma I\dot{\phi},$$

when

$$L_q = x_q Y_q - y_q X_q.$$

Since

$$\dot{x} = K_{x\theta}\dot{\theta}, \qquad \dot{y} = K_{y\theta}\dot{\theta} \qquad \text{and} \qquad \dot{\phi} = K_{\phi\theta}\dot{\theta}.$$

We have on substituting:

$$\Sigma X = \Sigma m K_{x\theta}\ddot{\theta} + \Sigma m\frac{dK_{x\theta}}{d\theta}\dot{\theta}^2,$$

$$\Sigma \bar{Y} = \Sigma m K_{y\theta}\ddot{\theta} + \Sigma m\frac{dK_{y\theta}}{d\theta}\dot{\theta}^2$$

and

$$\Sigma L = [\Sigma m(xK_{y0} - yK_{x0}) + \Sigma IK_{\phi 0}]\ddot{\theta} + \left[\Sigma m\left(x\frac{dK_{y0}}{d\theta} - y\frac{dK_{x0}}{d\theta}\right) + \Sigma I\frac{dK_{\phi 0}}{d\theta}\right]\dot{\theta}^2,$$

which are the equations for the determination of the overall or external reactions, including the reactions of all external constraints, in terms of the generalized coördinate of the mechanism for one degree of freedom.

CONDITIONS OF A DYNAMIC BALANCE AND THE REACTIONS ON THE FRAME OF A MACHINE.

The conditions of a dynamic balance of a machine are that the summations of the kinetic reactions of the moving parts and their moments are nil. These conditions of balance are usually assumed for a uniform velocity of the primary shaft, so that only the centripetal term of the reduced inertia forces with respect to the primary coördinate are included. Therefore,

$$\Sigma m\frac{dK_{x0}}{d\theta}\dot{\theta}^2 = 0, \qquad \Sigma m\frac{dK_{y0}}{d\theta}\dot{\theta}^2 = 0,$$

$$\Sigma m\left(x\frac{dK_{y0}}{d\theta} - y\frac{dK_{x0}}{d\theta}\right)\dot{\theta}^2 + \Sigma I\frac{dK_{y0}}{d\phi}\dot{\theta}^2 = 0,$$

$$\Sigma mz\frac{dK_{x0}}{d\theta}\dot{\theta}^2 = 0, \qquad \Sigma mz\frac{dK_{y0}}{d\theta}\dot{\theta}^2 = 0,$$

where z is the distance along the shaft from any arbitrary transverse plane. In estimating the angular velocity ratios in these equations, it is important to observe that the velocity ratios are assumed positive only when the rotation of a member is in the direction of the rotation of the primary crank.

In general, however, the degree of unbalance in a machine is manifested by a reaction on its frame. The reaction on the frame can be divided into five components, the two coördinate translation components in a plane transverse to the axis of the shaft, the torque reaction about the shaft and two moment reactions in planes at right angles and passing through the axis of the shaft.

In considering the reactions on the moving parts, it is important to include the reactions of connected parts when these parts are external to the frame of the machine. Such reactions would include belt pulls, the torsion moment of shafts, etc. Thus in a reciprocating engine in considering the torque reaction on the frame, we must include the torque on the shaft force on the moving parts:

Reaction on the Frame (Figs. 4 and 5):—

Let T_D = torsion moment on primary shaft

T_f = torque reaction on frame about primary shaft

T_R = resisting torque on driver or secondary shaft

X_D, Y_D = coördinate components of the bearing reaction of frame on driving or primary shaft

X_R, Y_R = coördinate components of the bearing reaction of frame on driven shaft.

The reactions on the frame for the four bar linkage mechanism, Fig. 4, consists of:

$$T_f = x_r \bar{Y}_r - y_r X_r$$

and

$$\Sigma X = X_r + X_d, \qquad \Sigma Y = Y_r + Y_d$$

and for the reciprocating engine, Fig. 5:

$$T_f = -N(l \cos \psi + r \cos \theta)$$

and

$$\Sigma X = P - X_d, \qquad \Sigma Y = Y_d - N,$$

where

l = length of connecting rod,

r = radius of crank,

ψ = connecting rod obliquity,

P = steam pressure on cylinder head.

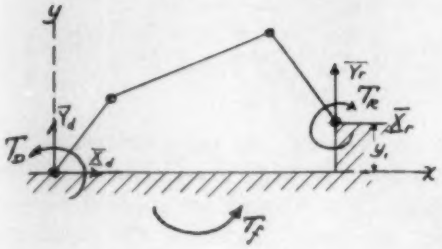


FIG 4

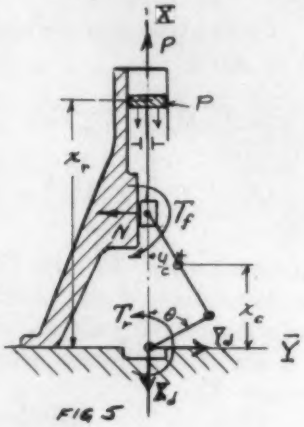


FIG 5

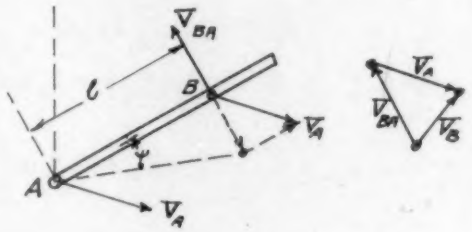


FIG. 6.

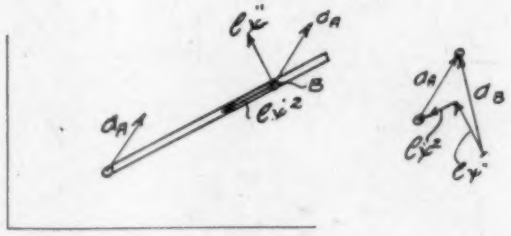


FIG 7.

From the generalized equations of motion, we have (Fig. 4):

$$T_D - T_R K_{\phi\theta} = [\Sigma m(K_{x\theta}^2 + K_{y\theta}^2) + \Sigma I K_{\phi\theta}^2]\ddot{\theta} + \left[\Sigma m\left(K_{x\theta}\frac{dK_{x\theta}}{d\theta} + K_{y\theta}\frac{dK_{y\theta}}{d\theta}\right) + \Sigma I K_{\phi\theta}\frac{dK_{\phi\theta}}{d\theta}\right]\dot{\theta}^2,$$

from which if T_d and T_r are known, we may calculate the accelerations of the mechanism, or with a constant angular velocity of the primary shaft ($\ddot{\theta} = 0$) and an applied torque T_D on the primary shaft, we may calculate the resisting torque T_R.

Then for the reactions on the frame (Fig. 4):

$$\Sigma T + T_f = T_d - T_r + T_f = [\Sigma m(xK_{y\theta} - yK_{x\theta}) + \Sigma I_{\phi\theta}K_{\phi\theta}]\ddot{\theta} + \left[\Sigma m\left(x\frac{dK_{y\theta}}{d\theta} - y\frac{dK_{x\theta}}{d\theta}\right) + \Sigma I\frac{dK_{\phi\theta}}{d\theta}\right]\dot{\theta}^2,$$

where

$$T_f = x_r Y_r - y_r X_r$$

$$\Sigma X = X_d + X_r = \Sigma m K_{x\theta}\ddot{\theta} + \Sigma m\frac{dK_{x\theta}}{d\theta}\dot{\theta}^2,$$

$$\Sigma Y = Y_d + Y_r = \Sigma m K_{y\theta}\ddot{\theta} + \Sigma m\frac{dK_{y\theta}}{d\theta}\dot{\theta}^2.$$

For the reciprocating engine mechanism: Fig. 5,

$$-PK_{x\theta} - T_r = [m_r K_{x,\theta}^2 + m_c(K_{xc\theta}^2 + K_{yc\theta}^2) + I_c K_{\varphi\theta}^2 + I_0]\ddot{\theta} + \left[m_r K_{x,\theta}\frac{dK_{x,\theta}}{d\theta} + m_c\left(K_{xc\theta}\frac{dK_{xc\theta}}{d\theta} + K_{yc\theta}\frac{dK_{yc\theta}}{d\theta}\right) + I_c K_\theta\frac{dK_\theta}{d\theta}\right]\dot{\theta}^2,$$

where

$$K_{x,\theta} = \frac{\dot{x}_r}{\dot{\theta}} = \frac{dx_r}{d\theta},$$

x_r being the coördinate for the reciprocating parts. We may

determine either the acceleration or the resisting torque for a given angular velocity of the shaft $\dot{\theta} = \omega$.

To calculate the reactions on the frame, we have:

$$T_f - T_r = [m_c(x_c K_{yc\theta} - y_c K_{xc\theta}) + I_c K_\theta + I_0]\ddot{\theta} + \left[m_c\left(x_c \frac{dK_{yc\theta}}{d\theta} - y_c \frac{dK_{xc\theta}}{d\theta}\right) + I_c \frac{dK_{\psi\theta}}{d\theta}\right]\dot{\theta}^2,$$

where $T_f = N(l \cos \psi + r \cos \theta)$

$$\sum X = P - X_d = - \sum m K_{x\theta}\ddot{\theta} - \sum m \frac{dK_{x\theta}}{d\theta}\dot{\theta}^2,$$

$$\sum Y = N - Y_d = \sum m K_{y\theta}\ddot{\theta} + \sum m \frac{dK_{y\theta}}{d\theta}\dot{\theta}^2.$$

Due to the linear nature of these equations, we may apply the principle of superposition and superimpose the efforts of each kinetic loading of a member or applied loading on the mechanism.

Thus for the torque reaction due to steam pressure, we have, assuming uniform angular velocity of the shaft:

$$- PK_{x\theta} - T_r' = 0, \qquad T_r' = - P\frac{x_r}{\dot{\theta}},$$

and

$$T_f' - T_r' = 0; \qquad \therefore \qquad T_f' = T_r' = - P\frac{\dot{x}_r}{\dot{\theta}}.$$

Again for the inertia moment of the rod, the corresponding reaction on the frame is:

$$- T_r'' = I_c K_{\psi\theta}\frac{dK_{\psi\theta}}{d\theta}\omega^2,$$

where $\dot{\theta} = \omega =$ constant and

$$T_f'' = T_r'' + I_c \frac{dK_{\psi\theta}}{d\theta}\omega^2;$$

$$\therefore \quad T_f'' = I_c \frac{dK_{\psi\theta}}{d\theta}(1 - K_{\psi\theta})\omega^2.$$

Now $K_{\psi\theta} = - \psi/\omega$ since the rotation of the rod corresponding

to an increase of θ is opposite to the positive rotation of the crank. Further:

$$\frac{dK_{\psi\theta}}{d\theta} = -\frac{\ddot{\psi}}{\omega^2},$$

so that

$$T_f'' = I_c\ddot{\psi}\left(1 + \frac{\psi}{\omega}\right),$$

a result that can readily be shown by a simple resolution of forces.

KINEMATIC PROCEDURE.

We will now consider methods for obtaining the velocity ratios and their derivatives for various machine combinations. Advantage will be taken of the graphical methods, i.e. velocity diagrams, etc., while the relations will be so far as possible expressed analytically. That is, we will proceed along a grapho-analytical method.

The determination of velocity ratios and their derivatives with respect to the fundamental coördinate of the mechanism may be obtained, (1) entirely analytically, (2) by the use of velocity diagrams expressed either in an analytical or graphical form, and (3) by the method of centers.

Following the method of Releaux, we may consider any mechanism as derived from a combination of elementary kinematic chains as the four bar linkage, the sliding block linkage, the swinging block linkage, etc. In the analytical procedure we can derive basic expressions for the velocity ratios and their derivatives for the motion of the members in each of these elementary combinations and thus a general extension may be made for any complicated mechanism.

With the use of velocity diagrams we have two general methods of resolution:

1. When the motion is compounded of a relative motion with respect to a moveable frame always maintained parallel to the fixed axis.

2. When the motion is compounded of a motion relative to a rotating frame with its origin subjected to a translation, so that the relative motion is compounded with the angular and translatory motion of the moveable frame relative to fixed axis.

When dealing with linkages without sliding pairs, it is generally of particular value to maintain the moving axis parallel to the fixed frame of reference. Since the angles and angular motions are then one and the same, whether referred to the moving or fixed axis, we may entirely suppress the moving axis and consider the motion relative to a point in the moving frame, say the origin. That is, the relative motion of B is compounded vectorially of the motion of B relative to A (the frame) and A relative to the fixed axis. Due to the rigidity of a member, the motion of B relative to A can only be due to the rotational displacement of B relative to A. That is, Fig. 6,

$$\bar{v}_B = \bar{v}_{BA} + \bar{v}_A = \overline{l\psi} + \bar{v}_A.$$

The acceleration B relative to A is compounded of a tangential component $l\dot{\psi}$ and centripetal component $l\psi^2$ so that for an acceleration of the origin, we have (Fig. 7):

$$\bar{a}_B = \overline{l\dot{\psi}} + \overline{l\psi^2} + \bar{a}_A.$$

With the sliding pairs, however, the analysis of the motion with respect to a rotating frame is of advantage.

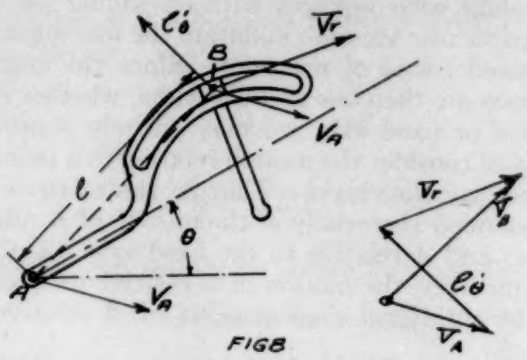
With rotating axis we compound the motion relative to the moving frame with the motion of the frame *at the point under consideration*, which latter includes the motion of the origin and the velocity at the point, due to the angular motion of the frame. The relative motion, such as the relative displacement of the block in the slide, introduces in the acceleration an additional component known as the "Coriolis" component or the complimentary acceleration. This is a distinctive feature when a relative motion is superimposed on a frame that rotates.

Thus consider the motion of a block B in a curved slot of radius r in a link member with axis at A (Fig. 8).
Then,

$$\bar{v}_B = \bar{v}_r + \overline{l\dot{\theta}} + \bar{v}_A,$$

while for the acceleration of the sliding block B (Fig. 9):

$$\bar{a}_B = \dot{v}_r + \left(\overline{2v_r\dot{\theta}} - \frac{\bar{v}_r^2}{r}\right) + \overline{l\ddot{\theta}} + \overline{l\dot{\theta}^2} + \bar{a}_A$$



FIGB

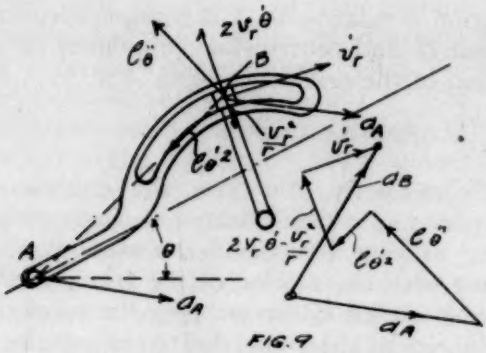


FIG. 9

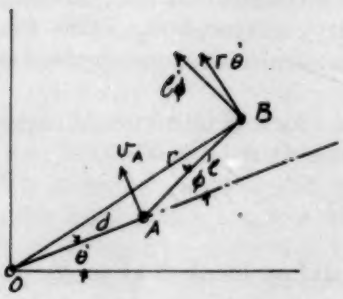


FIG 10

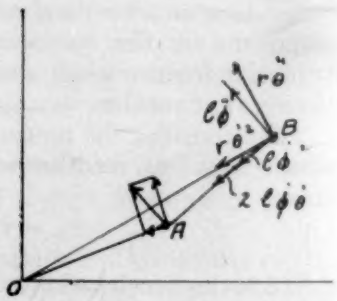


FIG.11.

where, it is to be noted, the Coriolis component $2v_r\dot{\theta}$ and the relative centripetal component v_r^2/r are normal to the slide, while $l\ddot{\theta}$ and $l\dot{\theta}^2$ are the tangential and radial acceleration components of the frame at B, due to its rotation about A, and $\dot{v}_r$ is the relative acceleration of the block B in the slot.

The angular velocity and acceleration of the block B is compounded of the relative angular motion for the curved slot and that of the frame or link; that is,

$$\dot{\psi}_B = \dot{\phi} + \dot{\theta} = \frac{v_r}{r} + \dot{\theta}$$

and

$$\ddot{\psi}_B = \ddot{\phi} + \ddot{\theta}.$$

When one member rotates relative to another, we may consider one member as the frame and the common joint as the axis of rotation and origin of the moving frame. The motion of any point in the second member in terms of the motion relative to the first or moving frame about the moving axis of rotation is:

For the velocities: Fig. 10,

$$\bar{v}_A = \overline{\dot{\theta} \cdot d}, \qquad \bar{v}_B = \overline{l\dot{\phi}} + \overline{r\dot{\theta}},$$

where $\overline{r\dot{\theta}} = \bar{v}_A + \overline{l\dot{\theta}}$, so that:

$$\bar{v}_B = l(\dot{\phi} + \dot{\theta}) + \bar{v}_A.$$

Obviously $\dot{\psi} = \dot{\phi} + \dot{\theta}$ is the absolute angular velocity with respect to parallel or fixed axis. It is to be noted that $\overline{l\dot{\phi}}$ is the relative and $\overline{r\dot{\theta}}$ is the motion of the frame at the point B.

For the accelerations (Fig. 11):

$$\bar{a}_A = \overline{\ddot{\theta} \cdot d} + \overline{\dot{\theta}^2 d},$$
$$\bar{a}_B = \overline{l\ddot{\phi}} + \overline{l\dot{\phi}^2} + \overline{r\ddot{\theta}} + \overline{r\dot{\theta}^2} + \overline{2l\dot{\phi}\dot{\theta}},$$

since,

$$\overline{r\ddot{\theta}} + \overline{r\dot{\theta}^2} = \overline{l\ddot{\theta}} + \overline{l\dot{\theta}^2} + \overline{\ddot{\theta} \cdot d} + \overline{\dot{\theta}^2 d}$$

and $\bar{l} + \bar{d} = \bar{r}$;

$$\therefore \quad \bar{a}_B = \overline{l\ddot{\phi}} + \overline{l\dot{\phi}^2} + \overline{l\ddot{\theta}} + \overline{l\dot{\theta}^2} + \bar{a}_A + \overline{2l\dot{\phi}\dot{\theta}}$$
$$= \overline{l\ddot{\psi}} + \overline{l\dot{\psi}^2} + \bar{a}_A,$$

where $\psi = \theta + \phi$ = absolute angle with respect to fixed or parallel axis.

These elementary resolutions enable us to construct velocity and acceleration diagrams for any type of mechanism with one degree of freedom.

The motion of any member of a linkage is completely determined when the vector velocities of any two points are known relative to another member or frame. This is equivalent to a rotation about an instantaneous center determined from the intersection of the perpendiculars to the velocities. We thus require three specifications, four quantities for the velocities, minus one quantity due to rigidity, or the two coördinates of the instantaneous center plus the angular velocity. As a part of the mechanism, the directions of the motions at the joints are determined by the constraints imposed by the other members, thus reducing the motion to a function of a single coördinate.

If A and B are the two joints, the motion is completely specified when given.

1. The magnitude and direction of vector v_A and the direction of v_B or vice versa.

2. The magnitude of the longitudinal motion along the rod, the angular velocity of the rod and the magnitude of one of the lateral components at A or B.

3. The instantaneous center of AB (relative to the frame) with a given angular velocity ω or the directions only of the velocities at A and B and the angular velocity.

The kinematic conditions due to rigidity are (Fig. 12):

$$v_A \cos \alpha = v_B \cos \beta,$$

while from similar triangles we note, the angular velocity given by:

$$\omega = \frac{\bar{v}_B - \bar{v}_A}{AB} = \frac{v_a}{IA} = \frac{v_B}{IB},$$

from which we obtain the relations:

$$\frac{v_A \sin \alpha}{v_B \sin \beta} = \frac{OA}{OB}$$

and

$$\omega = \frac{v_A \sin \alpha}{OA} = \frac{v_B \sin \beta}{OB} = \frac{v_A \sin \alpha + v_B \sin \beta}{AB}.$$

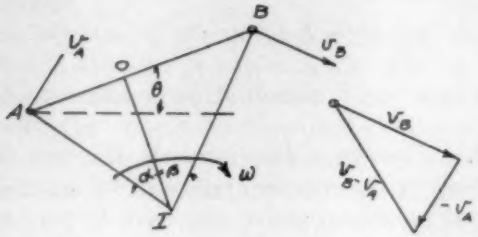


FIG 12

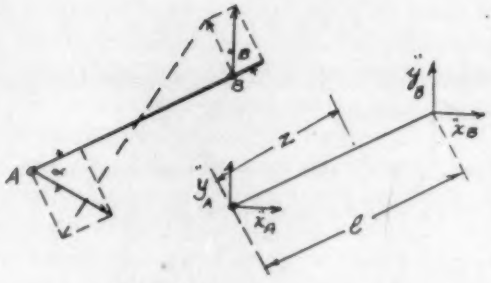


FIG 13

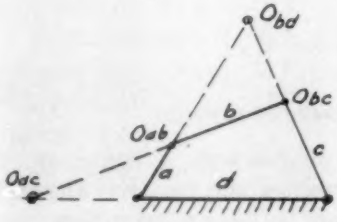


FIG. 14

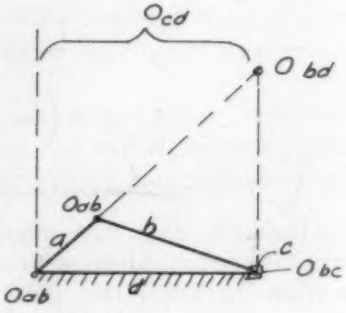


FIG. 15

For the accelerations, we have:

$$\bar{a}_B = \overline{l\ddot{\theta}} + \overline{l\dot{\theta}^2} + \bar{a}_A,$$

where $\dot{\theta} = \omega = (\bar{v}_B - \bar{v}_A)/l$ obtained from the velocity diagram.

Thus in combination with the velocity diagram and the condition of rigidity of the member, only three specifications of the accelerations at A and B are required.

The component of the acceleration along the rod are related by the equation (Fig. 13):

$$\bar{a}_A \cos \alpha - \bar{a}_B \cos \beta = \overline{l\dot{\theta}^2}$$

and the lateral or tangential components are (Fig. 13):

$$\bar{a}_B \sin \beta - \bar{a}_A \sin \alpha = \overline{l\ddot{\theta}}.$$

It is sometime convenient to resolve the accelerations into x and y components, then (Fig. 13):

$$\ddot{x} = \ddot{x}_A - (\ddot{\theta} \sin \theta_0 + \dot{\theta}^2 \cos \theta_0)Z = \ddot{x}_A + m_x Z,$$
$$\ddot{y} = \ddot{y}_A + (\ddot{\theta} \cos \theta_0 - \dot{\theta}^2 \sin \theta_0)Z = \ddot{y}_A + m_y Z,$$

where

$$m_x = \frac{\ddot{x}_B - \ddot{x}_A}{l} \quad \text{and} \quad m_y = \frac{\ddot{y}_B - \ddot{y}_A}{l}.$$

Therefore,

$$\ddot{x} = \left(\frac{l - Z}{l}\right)\ddot{x}_A + \frac{Z}{l_x}\ddot{x}_B,$$

$$\ddot{y} = \left(\frac{l - Z}{l}\right)\ddot{y} + \frac{Z}{l}\ddot{y}_B.$$

INSTANTANEOUS AND PERMANENT CENTERS.

Between any two members, or pairing, a common or mutual center exists, which determines the relative angular motion between the pair. When adjacent members are connected by a hinged joint, either moveable or fixed, we have a permanent center. All other centers between pairings are virtual or instantaneous, inasmuch as in general the

position of the center relative to the pair is a function of the configuration.

With a permanent center the virtual displacement of a point in one member relative to the other is perpendicular to the radius vector to the point from the center. With a sliding pair the virtual relative displacement is along the slide, and for the translatory displacements the virtual center is located at an infinite distance along the perpendicular to the line of action of the slide. If at any two points the direction of the vectors of the velocities at any instant is known, the virtual or instantaneous center of this member is at once determined from the intersection of perpendiculars to the velocities at any two points. The velocities are assumed relative to the fixed member or basic frame of the mechanism, i.e., the virtual center determines the instantaneous point of rotation of the member with respect to the fixed member. Since, however, we may regard the motion of rotation referred to any member as fixed, by inversion we may exchange our fixed reference member; the mutual center of rotation is the virtual or instantaneous center of the pair.

In the four bar linkage, Fig. 14, the virtual center of b relative to d or vice versa is O_{ad} and a relative to c is O_{ac}, while the permanent centers are O_{cd}, O_{cd} and O_{bc}.

In the sliding block linkage, Fig. 15, we have four elements or members: a, b, the sliding block c with permanent centers O_{ab}, O_{bc} and O_{ad}. The virtual centers are O_{bd}, O_{ac} and O_{cd} at infinity. Thus a relative to c consists of a motion at O_{ab} at right angles to b and O_{ad} in the direction of d and the perpendiculars to these displacements locating O_{ac} and vice versa for c relative to a.

For either mechanism we note a common center between any pairing. Now "a" pairs with b, c and d, while b pairs with c, d and a, etc. That is, the pairing combinations are represented by:

$$a(b + c + d),$$
$$b(c + d + a),$$
$$c(d + a + b),$$
$$d(a + b + c),$$

but as the center for ab and ba, bc and cb, etc., are respectively mutual, we have:

$$\frac{n(n-1)}{2} = \text{member of total centers,}$$

where n = total number of members, including fixed member.

An important property of virtual centers is the principle of three centers. Consider three members or bodies, a, which we will take as the reference frame with b and c (Fig. 16):

Obviously since the motion of O_{bc}, the common virtual center of b and c is both perpendicular to O_{ab}, O_{bc} and O_{ac}, O_{bc} respectively, then its motion relative to a being one and the same, the center O_{bc} can, therefore, only lie in the straight line O_{ab} and O_{ac}. That is, the centers of three adjacent members permanent and instantaneous, must always lie along a straight line.

As examples, in the four bar linkage. any three adjacent members as d, a and b, have their permanent or virtual centers O_{ad}, O_{ab} and O_{bd} along a straight line, or members, a, b, c, have their centers O_{ab}, O_{bc} and O_{ac} along a straight line. In the sliding block linkage, the permanent and virtual centers of d, a and b; that is, O_{ad}, O_{ab} and O_{bd}, must lie on a straight line O_{ad}, O_{bd}, while with adjacent elements, b, c and d, the permanent and virtual centers O_{bc}, O_{cd} and O_{bd} lie on the perpendicular to d through c.

A line of centers follow the notation $ar - rq - aq$, the order being indifferent. q cannot exceed the total number of members. Any unknown virtual center can at once be located by the intersection of any two lines of centers. Thus we have:

cf located by any two of $|cf - df|$ $|ce - ef|$ $|bc - bf|$ $|ac - af|$

be located by any two of $|bd - de|$ $|bc - ce|$ $|bf - ef|$ $|ab - ae|$,

where f is the total number of members equal 6 (a, b, $cdef$).

In general, we find for n members, $n - 2$ loci or lines of centers for each individual center. Since there are $\frac{1}{2}n(n-1)$

total centers, we have for the total number of possible lines of centers equal to:

$$\frac{n(n-1)(n-2)}{1\,2\,3} = \frac{1}{6}n(n-1)(n-2),$$

where the factor 3 takes care of the three mutually common centers per line.

It is to be particularly noticed that a permanent or virtual center gives merely the relative motion between any members; the principle of three centers extends the comparisons of motions to the motions of any two members relative to a third.

ACCELERATION COMPONENTS IN TERMS OF VELOCITY RATIOS AND THEIR DERIVATIVES.

In a system with one degree of freedom, the acceleration components of the members are readily expressed in terms of the velocity ratios and their derivatives with respect to the fundamental coördinate of the mechanism; that is,

For any angular coördinate ϕ:

$$\ddot{\phi} = K_{\phi\theta}\ddot{\theta} + \frac{dK_{\phi\theta}}{d\theta}\dot{\theta}^2, \qquad \text{where} \qquad K_{\phi\theta} = \frac{\dot{\phi}}{\dot{\theta}},$$

and for linear coördinate s:

$$\ddot{S} = K_{s\theta}\ddot{\theta} + \frac{dK_{s\theta}}{d\theta}\dot{\theta}^2, \qquad \text{where} \qquad K_{s\theta} = \frac{\dot{s}}{\dot{\theta}}.$$

Thus if we consider a part of any mechanism with one degree of freedom (Fig. 17):

$$\dot{x} = -r\dot{\theta}\sin\theta - l\dot{\phi}\sin\phi,$$

$$\dot{y} = r\dot{\theta}\cos\theta + l\dot{\phi}\cos\phi;$$

$$\therefore \quad K_{x\theta} = -r\sin\theta - l\sin\phi \cdot K_{\phi\theta},$$

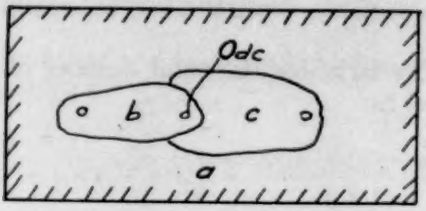


FIG.16

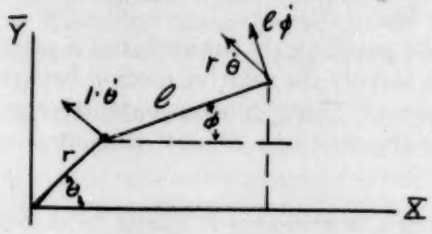


FIG 17.

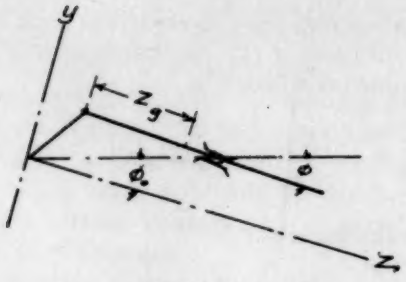


FIG 18

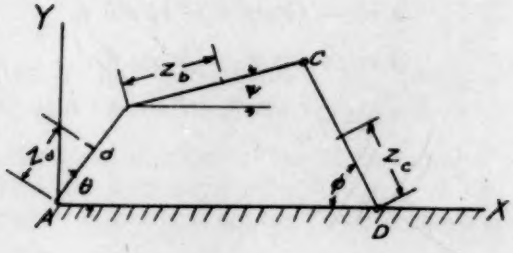


FIG.19.

where

$$K_{\phi\theta} = \frac{\dot{\phi}}{\dot{\theta}},$$

$$K_{y\theta} = r \cos \theta + l \cos \phi \cdot K_{\phi\theta},$$

$$\frac{dK_{x\theta}}{d\theta} = -r \cos \theta - l \cos \phi \cdot K_{\phi\theta}^2 - l \sin \phi \frac{dK_{\phi\theta}}{d\theta},$$

$$\frac{dK_{y\theta}}{d\theta} = -r \sin \theta - l \sin \phi K_{\phi\theta}^2 + l \cos \phi \frac{dK_{\phi\theta}}{d\theta},$$

where

$$\frac{dK_{\phi\theta}}{d\theta} = \frac{\ddot{\phi} - K_{\phi\theta}\ddot{\theta}}{\dot{\theta}^2}.$$

Therefore,

$$\ddot{x} = K_{x\theta}\ddot{\theta} + \frac{dK_{x\theta}}{d\theta}\dot{\theta}^2$$

$$= -r \sin \theta\ddot{\theta} - l \sin \phi\ddot{\phi} - r \cos \theta \cdot \dot{\theta}^2 - l \cos \phi \cdot \dot{\phi}^2,$$

$$\ddot{y} = K_{y\theta}\ddot{\theta} + \frac{dK_{y\theta}}{d\theta}\dot{\theta}^2$$

$$= r \cos \theta\ddot{\theta} - r \sin \theta\dot{\theta}^2 + l \cos \phi\ddot{\phi} - l \sin \phi\dot{\phi}^2,$$

which verifies the acceleration components.

For an oscillating mechanism (Fig. 18), a simplification is made by choosing our axis along and normal to the rod, so that the axis z makes an angle ϕ_0 and ϕ with the horizontal. In doing so, care must be given to maintaining the trigonometric functions, since we are concerned with their rates of changes.

Then,

$$\ddot{\phi} = K_{\phi\theta}\ddot{\theta} + \frac{dK_{\phi\theta}}{d\theta}\dot{\theta}^2,$$

where

$$K_{\phi\theta} = \frac{r}{z_c} \cos (\theta + \phi),$$

$$\frac{dK_{\phi\theta}}{d\theta} = -\frac{r}{z_c} \sin (\theta + \phi)(1 + K_{\phi\theta}) - \frac{r}{z_c^2} \cos (\theta + \phi) K_{z\theta}$$

and

$$K_{z\theta} = -r \sin(\theta + \phi).$$

For the motion of the c. of g. P of the oscillating rod, since,

$$z_p = r \cos(\theta + \phi_0) + z_g \cos(0),$$
$$\dot{z}_p = -r \sin(\theta + \phi_0)\dot{\theta} - z_g \sin(0)\dot{\phi};$$
$$\therefore \quad K_{zp\theta} = -r \sin(\theta + \phi_0) - z_g \sin(0) K_{\phi\theta} = K_{z\theta},$$
$$\frac{dK_{zp\theta}}{d\theta} = -r \cos(\theta + \phi_0) - z_g \cos(0) K_{\phi\theta}^2 = -r \cos(\theta + \phi_0) - z_g K_{\phi\theta}^2;$$
$$\therefore \quad \ddot{z}_p = -r \sin(\theta + \phi)\ddot{\theta} - r \cos(\theta + \phi)\dot{\theta}^2 - z_g \dot{\phi}^2,$$

since

$$y_p = r \sin(\theta + \phi_0) - z_g \sin(0),$$
$$\dot{y}_p = r \cos(\theta + \phi_0)\dot{\theta} - z_g \cos(0)\dot{\phi};$$
$$\therefore \quad K_{yp\theta} = r \cos(\theta + \phi) - z_g K_{\phi\theta},$$
$$\frac{dK_{yp\theta}}{d\theta} = -r \sin(\theta + \phi_0) - z_g \frac{dK_{\phi\theta}}{d\theta}$$

and since

$$\frac{dK_{\phi\theta}}{d\theta} = \frac{\ddot{\phi} - K_{\phi\theta}\ddot{\theta}}{\dot{\theta}^2},$$
$$y_p = r \cos(\theta + \phi)\ddot{\theta} - r \sin(\theta + \phi)\dot{\theta}^2 - z_g \ddot{\phi}.$$

(To be continued)

NOTES FROM U. S. BUREAU OF STANDARDS.*

CHANGE IN VOLUME DURING USE OF BURNED CLAY BODIES.

THE Columbus Branch of the bureau is conducting an investigation of the change in volume of burned clay bodies resulting from the autoclave treatment. In order to study the effect of composition of the body on the expansion caused by the autoclave treatment, several bodies with the following limits of composition were made and subjected to a three hour treatment at 150 lbs./in.^{2} steam pressure.

Clay (total)	48 to 56 per cent.
Flint	26 to 34 per cent.
Feldspar	14 to 18 per cent.

Expansions due to the autoclave treatment were taken on the bodies fired to cones 5–6, 8–9, and 11–12. It was found that practically a linear relationship existed between the percentage of expansion and the cone firing temperature. Results pointed that on replacing clay by feldspar and with the flint constant, there was an increase in expansion, although the change was little more than the experimental error involved in the measurements. Also on increasing clay at the expense of flint, with feldspar constant, a greater expansion is secured. The changes in expansion within the above body limits are small and seem to show that little can be done in the way of decreasing the expansion by a variation of the feldspar-clay-flint content providing the same raw materials are used and the bodies have the same ball clay-china clay ratio.

Bars made of typical English and American ball and china clays were fired to cones 5–6, 8–9 and 11–12 and their expansions determined after a three hour autoclave treatment under a steam pressure of 150 lbs./in^2. The two English china clays used gave a much greater expansion than did any of the other clays, with the two American china clays giving a considerably lower change. It was found that the bodies

* Communicated by the Director.

made of feldspar, clay, and flint gave a greater expansion than the pure clays which indicates that the changes are not due solely to the clay constituents of the body. The ball clays showed, with one exception, no expansion even at the cone 5–6 firing. One American ball clay showed an expansion of 0.014 per cent. at cone 5–6, but this decreased to zero at cone 8–9. It was found that with the same clay or body an increase in absorption would be accompanied by an increase in expansion.

Samples of commercial bodies made in the form of bars were subjected to a three hour autoclave treatment under a steam pressure of 150 lbs./in^2. and their changes in length noted. The absorptions of the bars after treatment were also determined. A buff wall tile body showed an expansion of 0.057 per cent. with an absorption of 6.6 per cent.; a cream wall tile expanded 0.016 per cent. and had an absorption of 2.1 per cent. A terra cotta body expanded from 0.045 to 0.056 per cent. with absorptions of 11.2 and 12.5 per cent. respectively. White semi-vitreous dinner ware expanded 0.071 per cent. (absorption 9.6 per cent.) and ivory semi-vitreous dinner ware expanded 0.065 per cent. (absorption 7.8 per cent.).

HEAT INSULATING MATERIALS.

WITHIN the past few years there has been great activity in the field of thermal insulation, particularly for dwellings, and a large number of materials have been utilized in the manufacture of insulating products of various kinds. There are numerous other raw materials, now considered more or less waste, which may be equally suitable for this purpose.

Anticipating the need for additional data, the bureau is conducting a study of the insulating properties of various fibrous materials. Tests have been made on jute, cotton, flax, wood fiber, bagasse, corn stalk fibers, moss, excelsior, wood shavings, etc. From an insulation standpoint, alone, there is practically no choice between the different commercial grades of these materials. The waste or poorest grades therefore may be commercially available for the manufacture of heat insulating materials.

Tests are being made of each fiber, packed to different densities ranging as widely as practicable, and it is found that each fiber has an optimum density at which the thermal conductivity is a minimum or the insulating value a maximum. This suggests that to derive the most insulating value from given amounts of material, it should be packed to its optimum density.

Tests have also been made on mixtures of different fibers and it has been found that the addition of relatively small quantities of good insulating materials to large quantities of comparatively poor material, may yield a product of surprisingly good insulation value.

EFFECT OF SMALL CHANGES IN TEMPERATURE ON THE PROPERTIES OF BODIES.

LABORATORY investigators who have used torsion wires for the accurate measurement of twisting forces, or diaphragms for the measurement of pressure, must have been puzzled at times by the question of determining what proportion of the observed change in stiffness of the elastic system with variations in temperature is due to a change of elastic modulus, and what proportion is caused by ordinary thermal expansion. Increase in the diameter of a torsion wire due to thermal expansion stiffens the wire but simultaneously there is a decrease in the rigidity or shear modulus of the material of the wire, tending to offset the influence of thermal expansion more or less.

In physical and engineering investigations involving the properties of bodies (as distinguished from substances or materials) it is often necessary to determine at least approximately what will be the effect of small departures from the normal working temperature. This is usually done either (1) by a detailed computation, or (2) by a direct experiment in which the temperature is actually varied.

A paper entitled "The Effect of Small Changes in Temperature on the Properties of Bodies," by N. D. Hersey, appears in the Bureau of Standards Journal of Research, Vol. 4, No. 1; January, 1930 (Research Paper No. 137). In this paper two new methods are given for determining the effect of small

departures from normal temperature upon any property of a body, such as the stiffness of a spring or the accuracy of an instrument: (1) a simple calculation, applicable in some cases, based on the theory of dimensions; (2) a combined mathematical and experimental solution, which eliminates the necessity for actually varying the temperature of the body, provided the thermal properties of the component material are known.

Finally it is pointed out that the same treatment can be extended to any other condition analogous to temperature, such as hydrostatic pressure.

The following illustrative examples are given: (1) Stiffness of a flat spring of uniform section; (2) Torsional stiffness of a wire of circular cross section; (3) Vibration frequency of a flat, circular diaphragm, clamped at the edge; (4) Period of a pendulum; (5) Stiffness of a spring of irregular shape subject to pure bending or twisting; (6) Electrical and thermal resistance; (7) Venturi air speed indicator; (8) Spring stiffness for complex stresses; (9) Vibration frequency of a loaded spring; (10) Coefficient of friction of a lubricated journal bearing; (11) Effect of pressure on electrical resistance.

MAIZOLITH.

AN entirely new type of material has recently been developed from cornstalks. This material is somewhat like hard rubber in appearance and properties.

It is believed that cornstalks may eventually find their way into the manufacture of insulating material, noiseless gears, and similar products.

Investigators who have tried to make paper pulp from cornstalks long ago recognized the difficulty of overcoming the extreme tendency to hydrate, which seems characteristic of this material. The invention of maizolith is an attempt to take advantage of this characteristic of cornstalks. It has been found rather easy by severe chemical and mechanical treatment to hydrate the cornstalk completely, so that the product is a jelly-like mass with no vestige of fibrous structure. Maizolith is prepared by drying this jelly and then machining the finished piece into the desired shape.

This development is a part of the coöperative work with Iowa State College to find industrial uses for cornstalks. A complete description of the process of manufacture will shortly be published by the bureau.

TENSILE PROPERTIES OF RAIL AND OTHER STEELS AT ELEVATED TEMPERATURES.

A PAPER on the tensile properties of rail and other steels at elevated temperatures will be presented by John R. Freeman, Jr., at the annual meeting of the American Institute of Mining and Metallurgical Engineers, in New York during the week of February 17. In this paper, it is pointed out that the Bureau of Standards has been coöperating over a period of years with rail manufacturers and railroads in a joint study to determine the origin of rail failures particularly the type known as transverse fissure. In some previous work the bureau made an extended study of the endurance properties of both new and used rails. The results of these studies led to the conclusion that the nucleus of the transverse fissure was probably present in the new rail before placing in track and that the fissure developed in service from this nucleus. The origin of this nucleus is obscure. It was thought to be a minute crack or group of cracks of microscopic dimensions which formed in the rail while hot at some time during its manufacture. A study has therefore been made of the strength and ductility of rail steels at elevated temperatures. It has been found that in the temperature range 500 to 700° C. the ductility of some rail steels and some other steels is relatively very low. This range of temperature has been termed the secondary brittle range and the phenomenon secondary brittleness. These terms have been given to distinguish the phenomenon from the well known phenomena of blue brittleness and hot shortness in steel.

A study was also made to determine the temperature distribution in a rail during cooling under different conditions.

As a result of these studies it has been shown that under certain conditions of cooling because of the large temperature gradients present the stresses developed in a rail or other steel structure while cooling through the secondary brittle range

may be sufficient to cause internal ruptures which may act as nuclei for fatigue failure (transverse fissures) in service.

SHOULD SIEVES COARSER THAN THE NO. 200 BE CALIBRATED?

THE appreciable differences in the performance of some sieves of the same nominal mesh, and coarser than the No. 200 sieve, have been brought to attention several times recently both through actual tests and in discussion between interested laboratories. It seems that consideration should be given to the feasibility and desirability of providing some means of calibrating these sieves. Sieves at least as coarse as the No. 20, while complying with the specifications for dimensions of openings, have been found to differ in results of sieving by very appreciable amounts. For example, careful hand tests on well prepared samples of sand, using similar methods for each sieve, gave the following as the indications of three No. 20 sieves, all of which sieves had been found in compliance with specifications.

Sieve	Percentage retained
A	6.3
B	3.4
C	2.0

These sieves have been used in other tests, with somewhat similar indications of relative performance. Three No. 30 sieves, when used in careful tests of another sample of sand, gave following indications:

Sieve	Percentage retained
D	3.8
E	7.3
F	7.7

A brief study of some other tests indicates that these differences in performance must be characteristic of some of the sieves used in the study of sands and other materials of similar type. There is the possibility that some work may not justify extreme refinement and that the labor of calibration might not be warranted by the benefits resulting therefrom. However, there are some cases where it is desired to obtain more nearly correct values, particularly when making

comparisons between various laboratories. This has been found true in foundry research. The abrasive industry, using sieves quite close together in the series of size openings, desires accurate indications of size and gradation of particles.

It has been understood that it is not feasible to make the tolerances for sieve mesh more stringent and, therefore, it would seem that one of the points meriting attention would be the practicability of providing standard samples, in somewhat the same manner as supplied for the calibration of the No. 200 sieve when used in testing cement. Such a question would require careful study of possibilities in the preparation of samples, the selection of standards with which to commence the work, the needs of interested industries and laboratories, and particularly whether the samples should be graded over a considerable range in sizes or be restricted to certain narrow zones of sizes, each of which zone might be applicable to certain sieves of materials.

Whatever the solution of the question, and the need of the refinement, it is evident that the careful testing of sieves for compliance with specifications for dimensions of wire and opening may result in a false idea of accuracy of the indications of such sieves so long as there is lacking a means of correcting them with respect to some definite standard of performance.

STANDARDIZATION OF GAGES.

A MEMBER of the gage section of the bureau attended the Chicago meeting of the American Petroleum Institute. At the meetings of the various standardization committees the general subject of gages and gaging practice was given considerable attention. At the meeting of the cable drilling tool joint committee the interchangeability of product passing the A. P. I. gages owned by the various manufacturers was the main subject discussed. Samples of the product of several manufacturers were exhibited and the interchange standoff of the samples was determined. In general this interchange was considered satisfactory although several discrepancies were noted which were difficult to account for. The samples of product have been shipped to the bureau for determination of lead errors. At the meeting of the pumping equipment com-

mittee samples of pin and star gages suitable for use as no go gages for working barrels were exhibited. The present solid cylindrical no go plug is not considered adequate. It was not believed advisable to require the use of a star or pin gage at the present time, but it was recommended that the manufacturers inspect barrels with one of the pin or star gage types exhibited, or a similar gage, during the coming year with the idea of adopting one type as standard next year.

The correlating committee on gages and gaging practice voted to no longer require the interchange of sucker rod go or no go gages with the grand masters deposited at the bureau. With certain changes in the specifications it was felt that the complete measurement of the dimensions of the gages, as is done at the bureau, was sufficient control of these straight threaded gages.

The question of the use of a mineral oil lubricant in assembling cable and rotary tool joint gages was discussed and it was decided that no change in the present practice of using medicinal mineral oil was desirable.

A member of the gage section also attended a meeting of the Diamond Core Drilling Manufacturers' Association at which the proposals of several gage manufacturers were discussed. It was found that the proposals differed somewhat with respect to the types and quality of gages to be furnished. Steps were taken to obtain bids on the types of gages recommended by the American Gage Design Committee.

INTERCHANGEABLE GROUND GLASS JOINTS FOR LABORATORY GLASSWARE.

ON December 17, a general conference of representative manufacturers and users of laboratory glassware met at the bureau and adopted a commercial standard for interchangeable ground glass joints. This standard, if accepted by the industry, should become the basis for every day trade in interchangeable joints, which will be marked "S. T.," indicating standard taper and also with the manufacturers' and distributors' trade mark, and the size.

These interchangeable joints will facilitate the setting up of laboratory apparatus and will provide for quick substitution of broken parts and effect considerable economies over the older method of having to replace the entire apparatus.

NOTES FROM THE U. S. BUREAU OF MINES.*

CENTRIFUGAL CONCENTRATION A PROMISING METALLURGICAL DEVELOPMENT.

AN investigation of the centrifugal concentration of certain types of ores, especially the application of the process to the treatment of the slime portion of tailings from mills that use gravity-concentration methods, has been conducted at the Rare and Precious Metals Experiment Station of the United States Bureau of Mines, Department of Commerce, Reno, Nevada, in coöperation with the University of Nevada. A centrifugal bowl designed by H. A. Doerner of the Bureau of Mines, gave encouraging results in preliminary tests on slime tailings from a mill concentrating a scheelite ore.

To construct and operate a centrifugal concentrator which will develop much more powerful centrifugal force than that used in the Bureau of Mines tests is entirely feasible, states Mr. Doerner in Technical Paper 457, just issued. The present machine concentrates and recovers particles that are too small to be saved by gravity machines, and there is no reason to doubt that a larger and more powerful centrifuge will effectively concentrate particles in a still lower range of sizes. The practical limit still remains to be determined.

The present work does not begin to exhaust the possibilities of centrifugal concentration. Gravity concentration requires a sized or classified feed for maximum results. The same is true for centrifugal concentration; but sizing of slimes is out of the question, and gravity classification is not effective in the lower range of sizes. Centrifugal classification, however, is possible and will be necessary to realize the full potentialities of centrifugal concentration. By the use of more countercurrent water or less speed of rotation with the same water the centrifugal jig described in the Technical Paper will operate as a classifier.

Jigs are primarily adapted to handling a coarse, sized feed. "Surface concentrators," such as tables and vanners are

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much more effective on fine, classified feed. It is therefore probable that a centrifuge employing surface concentration would concentrate slimes better than the present jig type of centrifuge.

The Bureau of Mines has recently constructed a centrifugal bowl that employs surface concentration in a manner exactly like that of an advanced type of slime table. Preliminary tests have given encouraging results.

There seems to be nothing to prevent the development of many types of centrifugal concentrators corresponding to the various gravity machines, such as jigs, tables, and classifiers. Although concentration of slime is the most promising field, the centrifuge will also concentrate fine sands.

Any competition with gravity machines in the latter field must be based upon capacity rather than recoveries or grade of product.

Testing a centrifugal concentrator involves unusual difficulties because it is not possible to observe adequately the concentrating action. The reason for any result must be deduced from theoretical considerations. Large machines are expensive, and their construction must be justified by results obtained in small experimental units. It is difficult, if not impossible, to obtain as good results in a small machine as in a large one, and it is hard to devise satisfactory details of design and operation for a small unit; therefore progress has been slow and failures frequent.

The Bureau of Mines believes that centrifugal concentration should be tested in the following fields:

1. Concentration of heavy minerals from the slime tailings or reground middlings of mills using gravity concentration.

2. Elimination of colloids from a crushed ore as a preliminary to flotation or leaching operations. The presence of excessive colloids in a milled ore may cause difficulties in, and greatly reduce the capacity of, plants using flotation or leaching operations. By passing the slime portion of such an ore through a centrifugal concentrator it may be possible to eliminate the colloids with only a small loss of valuable mineral and to obtain a colloid-free concentrate, much richer and of smaller bulk than the original slime. In a problem of this type the centrifuge should be adjusted to give a low ratio

of concentration, thereby obtaining the maximum recovery. The increase in plant capacity obtainable by such elimination of slime is worthy of considerable study.

3. Treatment of ores that slime badly. There are many known deposits of ore that are not amenable to flotation and that slime so badly when milled that gravity concentration will not obtain recoveries large enough for profitable operation. In many instances centrifugal concentration of the slimes may add enough to the recoveries to allow a profit.

4. Centrifugal concentration of slimes from the tailing dumps of gravity concentration mills.

5. Cleaning and dewatering of flotation concentrates and possibly the separation of lead and zinc from flotation concentrates.

6. Recovery of flour gold, tellurides, and amalgam from the tailings of amalgamation mills.

FLOW OF GASES THROUGH BEDS OF BROKEN SOLIDS.

ALTHOUGH a number of the most important industrial processes depend upon efficient contact between a bed of broken solids and a stream of moving gas or vapor, up to the present time very little work has been done on the laws that govern the flow of fluids through such beds. If new apparatus is built, old apparatus changed, or the mechanism of a process involving the above system altered, there are no means whereby a designer or engineer can predict the amount of pressure that will be necessary to force a given amount of gas through a given bed of broken material. Further, he has no definite means of ascertaining what arrangement of materials in the beds will give him the greatest efficiency of contact between gas and solid.

Information of this type is especially applicable to the detailed study of the physical and chemical reactions taking place within the iron blast furnace.

The North Central Experiment Station of the United States Bureau of Mines, Minneapolis, Minn., has undertaken a laboratory study of blast-furnace phenomena with the intention of eventually formulating a more or less complete quantitative theory of the physical and chemical reactions

within the furnace. The first phase of this work has been the laboratory study of the flow of gases through beds of broken solids. In so far as possible the results of the work have been formulated into general laws, but special attention has been paid to the particular materials entering into the blast-furnace processes.

The data so far obtained from this study have been published in Bulletin 307, by C. C. Furnas.

Two hundred and fifty-eight separate sets of data have been obtained and correlated in this work. In all instances the data have taken the form of the pressure required to force gas through a given system under given conditions.

Definite laws have been formulated that make it possible to predict with a reasonable degree of accuracy the resistance to gaseous flow of a bed of broken solids of sized material. It has not been possible to formulate general laws for systems involving all mixtures of sizes, but many data have been taken and recorded for such systems and compiled into rules of somewhat limited application.

The materials used have ranged in size from minus 100-mesh dust to pieces 25 centimeters in diameter. Data have been taken on lead shot, various iron ores, limestone, coke, and roll scale. Most runs have been made with air: a few data have been taken on hydrogen and carbon dioxide. Temperatures have ranged from 20° to 660° C.

These data are to be correlated in a later publication with data obtained for gas flow in commercial furnaces, thus affording a means of intelligently criticizing present blast-furnace practice and of suggesting and criticizing innovations.

CALCINATION OF TEXAS POLYHALITE.

THE behavior of Texas polyhalite (a complex salt composed of potassium, magnesium, and calcium sulfates with 6 per cent. water of crystallization) upon calcination is one of the important factors involved in several processes for the extraction of potash from this material. A detailed investigation of the calcination of polyhalite has been made by the Non-metallic Minerals Station of the United States Bureau of Mines, Department of Commerce, in coöperation with Rutgers University at New Brunswick, N. J.

Polyhalite dissolves very slowly in water and requires a considerable volume of water for complete extraction of the potash salts. If, however, finely ground polyhalite is first calcined so that most of its water of crystallization is driven off, it is possible to obtain more than 90 per cent. extraction of the total potash by using 2.3 parts of boiling water. Upon heating powdered polyhalite, vigorous evolution of steam is observed at 300° C. and about 50 per cent. of the total water present is given off between 300° and 310° C. Subsequent heating for one-half hour in the temperature range between 430° and 450° C. results in the removal of an additional 48 per cent. of the water. The remaining 2 per cent. is tenaciously retained and requires a higher temperature or prolonged heating for its removal.

The efficiency of the extraction of potash from calcined polyhalite (using boiling water), depends upon practically complete desiccation without sintering. Sintering is observed to start at about 475° C. and becomes very obvious at 500° C. while at 550° C. the originally powdered polyhalite sinters forming a hard crystalline mass. This sintered material yields its potash to boiling water very much more slowly than the material calcined below 450° C.

During calcination in the temperature range 300°–475° C. polyhalite is decomposed leaving an amorphous mixture of potassium, magnesium, and calcium sulfates. Above 475° C. and rapidly between 500° and 575° C., this amorphous powder sinters, forming a new compound, which is probably langbeinite (a double salt of potassium and magnesium sulfates, the calcium sulfate being held in solid solution in this new material). These changes are accompanied by marked variations in the refractive index, the determination of which may be used as a rapid method of controlling the calcination conditions.

MICROSCOPIC INVESTIGATIONS OF COMPLEX ORES.

MICROSCOPIC investigations of complex ores conducted at the Intermountain Experiment Station of the United States Bureau of Mines, Department of Commerce, Salt Lake City, Utah, in coöperation with the University of Utah, show that the microscope has an almost unlimited field of application

to the various phases of ore-dressing problems. During the progress of this work, the microscope has been applied as a fact-finding instrument to determine the mineralogical and other physical characteristics of numerous complex sulphide and oxide ores. Since the amenability of these ores to flotation or other treatment depends primarily upon a knowledge on the part of the operator of their physical structure, the data supplied by the microscope are invaluable.

The application of the microscope to ore-dressing problems in general is not restricted to the study of ores but is equally valuable as a fact finder in the study of concentrates, tailings, residues from leaching operations mattes and slags, in fact, practically every metallurgical product whose physical condition has a bearing on the cost or process involved in producing it.

Microscopic studies conducted at the Intermountain Experiment Station have shown that tailing losses in treating complex ores may be due to a variety of causes such as: Failure to grind sufficiently fine to liberate the valuable minerals, tarnished or oxidized films on the mineral surfaces, contamination of the surfaces of the mineral particles with clay, iron oxide and the slime products of rock decomposition and the presence of rare or uncommon minerals, whose existence was not suspected and no special provision made for their recovery.

Studies made of residues resulting from the leaching of low-grade copper ore have shown that the extent of fracturing in the ore and the relation of the mineral grains to the fractures are important factors in the time required for leaching. Since solutions penetrate rapidly through these channels and thus assist in the permeation of the rock, contiguous mineral particles are attacked more readily and isolated or disseminated particles require a much longer time of treatment.

Mill operators and persons engaged in testing work or the treatment of ores can not afford to neglect the microscope since it has been proven of constant value in the solution of ore-dressing and metallurgical problems.

THE FRANKLIN INSTITUTE.

ANNUAL MEETING, WEDNESDAY, JANUARY 15, 1930.

THE annual meeting of The Franklin Institute was called to order at eight-fifteen, by the President of the Institute, who presided.

The President announced that the meeting was the regular annual meeting of the Institute and called upon the Secretary for a statement of the business of the evening.

The Secretary announced that the minutes of the December meeting were printed in full in the January number of the Journal of the Institute, and moved that the minutes be approved as printed. The President announced that if there were no objection the minutes would be approved. No objection was offered and the minutes were declared by the President to be approved.

The Secretary presented the report of the Tellers of the election for officers and members of the Board of Managers which showed that the following gentlemen had been elected to the offices indicated: For President (to serve one year) Nathan Hayward; Vice-President (to serve three years) Walton Forstall; Treasurer (to serve one year) Benjamin Franklin; Managers (to serve three years) Charles E. Bonine, Edward G. Budd, Walton Clark, Charles Day, Clarence A. Hall, George A. Hoadley, James S. Rogers, and Haseltine Smith. He then stated that inasmuch as the President had been re-nominated and re-elected he, the Secretary, would announce that the election was a legal one and that the gentlemen whose names had been read were elected to the offices indicated.

The President expressed the appreciation of the Institute to the Tellers for their services in supervising and reporting the election.

The Secretary called to the attention of the meeting the special meeting of the Institute to be held on Wednesday, January twenty-second, at four-thirty p.m., for the consideration of the proposed sale of Institute property.

The President then read a synopsis of his annual report for the year 1929. The report will be published in full in the Year Book of the Institute.

There being no further business, the President introduced as the speaker of the evening Mr. Paul Shoup, President of the Southern Pacific Company of San Francisco, who spoke most interestingly on "Transportation Developments on the Pacific Coast." The speaker gave a historical review of the origins and developments of transportation, particularly in California, and dealt with the developments of the transcontinental lines which cross the United States. At the end of his talk Mr. Shoup gave a summary of present conditions of transportation on the Pacific Coast. The talk was beautifully illustrated by moving pictures of some of the great railroads and railroad sceneries of the Pacific Coast.

After the adjournment of the meeting, other pictures were shown to those who wished to remain. The meeting adjourned at ten p.m., with an expression of hearty appreciation to the speaker for his kindness in coming to us and for his delightful talk.

HOWARD MCCLENAHAN,
Secretary.

COMMITTEE ON SCIENCE AND THE ARTS.

(Abstract of Proceedings of Stated Meeting held Wednesday, January 8, 1930.)

HALL OF THE INSTITUTE,
PHILADELPHIA, January 8, 1930.

DOCTOR THOMAS D. COPE, *in the Chair.*

The Standing Committee on the Franklin Medal presented its report for final reading making the following recommendations, which were adopted:

That two Franklin Medals be awarded in 1930—one to Sir William Bragg, of London, England, "In recognition of a life-work in the study of x-rays and radioactivity, in the course of which he made fundamental contributions to that realm of physics, of his development of a method of determining molecular and crystal structure by the reflection of x-rays and of his fruitful guidance of the Davy-Faraday Research Laboratory and the Royal Institution of Great Britain";

and the other to

Doctor John Frank Stevens, of New York City, "In recognition of his unifying solutions of widely varying and difficult engineering problems met in the planning of the great Panama Canal, of the marked power shown by him in the organization of the engineering forces which later built that Canal, and of his eminent success in the location, construction and administration of railroads in this country and in foreign lands."

The following subjects were presented for first reading:

No. 2848: Di-Mold Method of Making Castings in Permanent Molds.
No. 2900: Bailey Meters.
No. 2918: Literature.

GEO. A. HOADLEY,
Secretary to Committee.

MEMBERSHIP NOTES.

ELECTIONS TO MEMBERSHIP.

(Stated Meeting, Board of Managers, January 15, 1930.)

RESIDENT.

MR. JULIAN T. BALDWIN, Chemist, Sandura Company, Inc., Paulsboro, New Jersey. For mailing: P. O. Box 154, Paulsboro, New Jersey.

MR. FENTON J. FITZPATRICK, Student, St. Joseph's College, Philadelphia, Pa. For mailing: 124 West Chestnut Hill Avenue, Philadelphia, Pa.

MR. CARROLL HODGE, Electrical Engineer, Electric Storage Battery Company, Philadelphia, Pa. For mailing: Radnor, Pa.

MR. HENRY W. PFEFFER, Engineer, 612 Otis Building, Philadelphia, Pa.

MR. EDWARD RICE, JR., Vessel Agent, 403 Bourse Building, Philadelphia, Pa. For mailing: 5301 Old York Road, Philadelphia, Pa.

MR. FRED L. JAHN, President, Watson and McDANIEL Company, Noble Street, west of Sixth, Philadelphia, Pa.

NON-RESIDENT.

MR. DAVID DIETZ, Science Editor, Scripps-Howard Newspapers, The Cleveland Press Building, Cleveland, Ohio.

MR. MAXWELL W. METZNER, Electrical Engineer, General Electric Company, Erie, Pa. For mailing: 1853 East Lake Road, Erie, Pa.

CHANGES OF ADDRESS.

DR. EDWARD G. ACHESON, 654 Madison Avenue, New York, City.

MR. E. R. BEATH, 201 South Twelfth Street, Philadelphia, Pa.

MR. RICHARD L. BINDER, President, Metals Coating Company of America, 497 North Third Street, Philadelphia, Pa.

MR. CHESTER L. DAVIS, 218 North Darling Street, Angola, Indiana.

MR. WILLIAM DEKRAFFT, 1790 Broadway, New York City.

DR. E. A. ECKHARDT, Gulf Research Laboratory, 327 Craft Avenue, Pittsburgh, Pa.

MR. FRED B. KAUFMAN, Hotel Kaufman, Towanda, Pa.

MR. FRED J. MILLER, Box 57, Lake Worth, Florida.

MR. O. W. A. OETTING, Globe-Union Manufacturing Company, 14 Keefe Avenue, Milwaukee, Wisconsin.

MR. HASELTINE SMITH, 127–129 South Fifth Street, Philadelphia, Pa.

MR. ALFRED O. TATE, 18 Summerhill Gardens, Toronto, Ontario, Canada.

MR. VICTOR W. ZILEN, 696 Richmond Avenue, Buffalo, New York.

NECROLOGY.

Mr. Wilfred Lewis, Haverford, Pa.

Mr. James Watson, Philadelphia, Pa.

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BOOK REVIEWS.

THE RAYON INDUSTRY. By Moïs H. Avram, B.Sc., M.E. Second Edition. 893 pages, illustrations, 8vo. New York, D. Van Nostrand Company, Inc., 1929. Price, $12.00.

A full notice was given of the first edition of this book and little remains to say of the present issue except that like its predecessor, it contains a very large amount of information concerning the composition, manufacture and marketing of artificial fibers, which have acquired in a few years a most important position in the textile field. The four types of products are treated in detail, viscose being the dominant product. A historical note gives accounts of the struggles of the early workers in the field. Such accounts are now-a-days interesting since, in the rapid progress of applied science, similar stories are always in the making. In earlier days, the artificial fiber was a nitrated product, but at present the bulk of material in the market is pure cellulose. Viscose is of this composition, but is obtained through transformation of cellulose xanthate. The details of manufacture of the artificial fibers are set forth fully with many tables relating to marketing and grading. An opportune note is given showing the troubles Congress has had in imposing a satisfactory duty on imported materials.

In the notice of the first edition the reviewer felt obliged to criticize unfavorably some features of the book. It was over-illustrated, large photogravures having no value in the purpose of the book not only add directly to cost, but require the use of a highly surfaced paper, increasing cost indirectly. The literary style was slipshod and often involved. Ordinary rules of grammar were often disregarded. "Data is" seems to be the only form the author knows. The printer's proof-reader might have caught this error. It was hoped that some attention would be paid to these criticisms, but the financial success of the book has caused author and publisher to receive the suggestions in the manner that Nicholas Koorn received the proclamation of William the Testy, read by Anthony the Trumpeter before the fortress of Bensellaerstein. Well, if Ephraim is determined to be joined to his idols, let him go.

HENRY LEFFMANN.

THE FUNDAMENTALS OF RADIO. By R. R. Ramsey, Ph.D., Professor of Physics, Indiana University, Member Inst. Radio Eng.; Fellow Am. Phys. Soc.; Fellow A. A. A. S.; Fellow Ind. Acad. Science, Author, "Experimental Radio." xi–372 pages, illustrations, 8vo, cloth. Bloomington, Ramsey Publishing Company, 1929. Price, $3.50.

EXPERIMENTAL RADIO. By R. R. Ramsey. Third Edition, enlarged. xii–255 pages, illustrations, 12mo, cloth. Bloomington, Ramsey Publishing Company, 1929. Price, $2.75.

By far the largest number of devotees to radio science are those who must approach the subject with the least possible mathematical attainment, and there

is no doubt that an understandable explanation of related phenomena, which generally are presented in mathematical language, can be made by the judicious use of descriptive ingenuity with the aid of elementary mathematics. The reader who has yet to complete his mathematical studies and seeks applicable knowledge of radio theory will find this book admirably suited to his purpose. Others as well, who have gone further in mathematical preparation, can wisely adopt it as a sound and profitable introduction, however farther they may desire to proceed. It may be mentioned that a subject of this kind, which deals with variables and their functions, necessarily entails the exercise of mathematical faculty, and those who are capable of mastering a treatise of this kind can, time permitting, qualify in the mathematics of customary texts on electrical theory. Unfortunately, "time" is often a most vexing item in that specification. Here and there the author does introduce a little calculus, but that need not deter the reader who is unacquainted with that subject from a perusal of the book since in the few instances in which it is used, he may accept the results derived by its use. Moreover those sections are developed and explained in a way helpful to the non-mathematical reader.

After a short account of the elements of electricity and magnetism, the reader is prepared for the subsequent chapters which deal specifically with radio and its multiplicity of devices of operation and control which at least by name have become familiar through widespread commercialization and use. The discussions are thorough, well illustrated and precise, and the serious student will be amply repaid for the effort required in mastering the analytic features which accompany the extensive descriptive matter of this book.

Laboratory manuals have become a familiar adjunct in all branches of experimental physical science, a helpful map of the territory to be covered with information as to procedure and a bit of theory here and there. Doctor Ramsey has collected the results of a long experience in his laboratory course in radio—that is, long as measured by the age of modern radio—into a laboratory manual of representative tests and other information of the actualities of the subject.

In the one hundred and twenty-eight experiments which are scheduled, all the usual measurements on inductance, capacity resonance, tube characteristics and other components which enter into the ensemble of the circuits of radio apparatus are adequately described. It is a most satisfactory collection of experimental data and method. With this volume and the one above noted, Doctor Ramsey has done much to place within the reach of all who have the will to learn the means to proceed.

L. E. P.

MECHANICAL EQUIPMENT OF BUILDINGS. By Louis Allen Harding, D.S., M.E., and Arthur Cutts Willard, S.B. Volume 1, Heating and Ventilation, a reference book for engineers, architects and contractors. Second edition, revised and enlarged. x–963 pages, illustrations, 8vo, imitation leather. New York, John Wiley & Sons, Inc., 1929. Price, $10.00.

Literature in all branches of technology has kept pace in volume and diversification with the tremendous strides of industrial development which we have witnessed in the course of the last three decades, though heating and ventilation seems to have lagged a bit in the early years. Prior to the nineties, there was little

information in book form on the technique of heating and ventilation engineering as it is understood today, and the designer of relatively small installations for buildings considered large in that day had for a guide such books as Thomas Box's Treatise on Heat which was based upon the old classic of Peclet, and several others probably patterned after it with additions of a practical character. But that branch of engineering shortly found its stride in harmony with the rapid pace of progress. Usable data which appeared in technical publications and in the transactions of engineering societies, notably the American Society of Heating and Ventilating Engineers gradually accumulated, and in 1895 the work of Professor Carpenter of Cornell University well represented the practice and requirements of that time.

The authors have undertaken an immense task in the preparation of a series of volumes on the mechanical equipment of buildings of which this, the first, volume deals with heating and ventilation. The work is intended for the two-fold purpose of providing engineers and architects with sufficient theoretical and commercial data for practical use in the designing room and also to serve as a text for students who desire to enter very fully into the subject. The first edition appeared in 1916 and although reprinted since, the authors have deemed a complete revision necessary to bring it into agreement with the present state of the art.

Manufacturing data are essential in the design of any project, and the inclusion of a considerable amount of such matter with illustrations, and properly classified, is most helpful in relieving the reader of a tedious search through manufacturers' literature for typical examples. Fundamental data on physical units, heat measurement, water, steam, air, fuels and combustion are contained in the opening chapters. While this information is to be found or should be found in every text dealing with the production and utilization of heat its presence is justified in a work which aims to give an exhaustive account of current practice. "Encyclopædic" may properly describe the character of the book. Every type of heating system in modern practice of heating large buildings is described, illustrated and analyzed, accompanied with all essential details of auxiliaries, connections and layouts. This extensive collection covers, steam, hot-water, hot-air, blower systems and the restricted but important field of electric heating.

Fan-systems are a comparatively old institution in large heating installations, but now, thanks to the general availability of electric current, artificial draft may conveniently be applied wherever desirable even in small installations for domestic heating. The "old-timers" who recall their struggles to derive adequate warmth from hot-air furnaces will be interested in the discussion on the rational design of the warm-air furnace heating systems and particularly in the account of fan furnace-heating systems in which the circulation of the heated air is stimulated by an electric fan. Small fans or blowers in domestic service have another application which is not specifically mentioned; that is their use, thermostatically controlled, with a closed ash-pit for burning "buckwheat" coal—the non-clinkering variety. Numerous arrangements to lighten the task and cost of house heating have been tried and have never been widely adopted, but this recent simple affair—an electric blower the size of one's hat in the cellar and a thermostatic switch in a room above seems to be gaining ground and, at the present price of "buckwheat," reducing the coal bill. The more elaborate systems of automatic temperature, and also moisture control, are duly considered, including theoretical

and applicable data on air conditioning, washing, humidifying, cooling, drying and filtering.

The section on heat transmission through the walls of buildings is especially full, coefficients being given for many types of construction, and the designer who requires the close estimate essential in planning large work may feel on safe ground in using these values, the sources of which are duly recorded. In connection with references; the addition of a formal bibliography which is omitted, would add materially to the value of the book for those readers who may desire to stress the analytical side of the subject.

The book is a well arranged compendium of all that pertains to modern practice in heating and ventilation. L. E. P.

QUANTUM MECHANICS. By Edward U. Condon, Ph.D., Professor of Theoretical Physics, University of Minnesota, and Philip M. Morse, Ph.D., Porter Ogden Jacobus, Fellow in Physics, Princeton University. xiii–250 pages, first edition. McGraw-Hill Book Company, Inc., New York, 1929. Price $3.

"Quantum Mechanics" is one of the volumes that comprise the International Series in Physics. Two other members of this series are, "Atoms, Molecules and Quanta" by Ruark and Urey and, Goudsmit and Pauling's, "The Atomic Model and the Structure of Line Spectra."

In an address before the American Association for the Advancement of Science, Professor P. W. Bridgman of Harvard remarked that: "Many of us could, I believe, confess to a feeling of breathlessness at the rapid changes of our present physical progress. . . . The upsetting feature here is not so much that we have discovered an enormous array of new facts, which in themselves are difficult enough to keep pace with, as that these facts have proved in many cases to be irreconcilable with our previous expectations of what was possible, so that we have been forced to change our entire conceptual attitude. These conceptual changes have in many cases been associated with mathematical theories which are being continually formulated at an ever-accelerating tempo and in a complexity and abstractness increasingly formidable." It is to be hoped that this book will be the forerunner of many of a similar type, the object of which is to evolve a semblance of order from the seemingly unending variety of mathematical formulations leading to the conception and developments of the principles of "wave mechanics." In the words of the authors: "The book is, rather, intended to be an outline of the results obtained by the quantum mechanics, and a manual of various methods which are used in arriving at these results. An attempt has been made to standardize the notation, to unify the treatment of normalizing factors and of the polynomials, and to correlate the various exact solutions of the Schrödinger equation, so that the book can be used as a reference work when dealing with any of the known wave functions."

Taken in order, the chapter titles are: Simple Mechanical Problems by the Wave Method; Radiation and External Fields of Simple Mechanical Systems; Approximate Methods of Handling the Wave Equation, Perturbation Theory; Diatomic Molecules; General Formulation of the Theory; Aperiodic Phenomena. As an addition to the chapter on Aperiodic Phenomena the authors mention a report on Collision Processes which is to appear in the January 1930 issue of the *Physical Review* Supplement. T. K. CLEVELAND.

NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS.

Report No. 323. Flow and Force Equations for a Body Revolving in a Fluid. In five parts. By A. F. Zahm. 41 pages, illustrations, quarto. Washington, Government Printing Office, 1929. Price, twenty cents.

This report, submitted to the Committee for publication, is a slightly revised form of U. S. Navy Aerodynamical Laboratory Report No. 380, completed for the Bureau of Aeronautics in November, 1928. The diagrams and tables were prepared by Mr. F. A. Louden; the measurements given in Tables 9 to 11 were made for this paper by Mr. R. H. Smith, both members of the Aeronautics Staff.

Part I gives a general method for finding the steady-flow velocity relative to a body in plane curvilinear motion, whence the pressure is found by Bernoulli's energy principle. Integration of the pressure supplies basic formulas for the zonal forces and moments on the revolving body.

Part II, applying this steady-flow method, finds the velocity and pressure at all points of the flow inside and outside an ellipsoid and some of its limiting forms, and graphs those quantities for the latter forms. In some useful cases experimental pressures are plotted for comparison with theoretical.

Part III finds the pressure, and thence the zonal force and moment, on hulls in plane curvilinear flight.

Part IV derives general equations for the resultant fluid forces and moments on trisymmetrical bodies moving through a perfect fluid, and in some cases compares the moment values with those found for bodies moving in air.

Part V furnishes ready formulas for potential coefficients and inertia coefficients for an ellipsoid and its limiting forms. Thence are derived tables giving numerical values of those coefficients for a comprehensive range of shapes.

Report No. 329. The Torsional Strength of Wings. By C. P. Burgess. 14 pages, illustrations, quarto. Washington, Government Printing Office, 1929. Price, ten cents.

This report is submitted to the Committee by the Bureau of Aeronautics, Navy Department. It describes a simple method for calculating the position of the elastic axis of a wing structure having any number of spars. It is shown that strong drag bracing near the top and bottom of a wing greatly increases the torsional strength. An analytical procedure for finding the contribution of the drag bracing to the torsional strength and stiffness is described, based upon the principle of least work, and involving only one unknown quantity.

The validity of the new method of analysis is tested by applying it to a two-fifths scale model of the large steel tubular 3-spar wing of the Huff-Daland XHB monoplane. The calculated stresses are checked by comparison with the strains observed by means of electric telemeter strain gauges secured to the spars during sand load tests in the static testing laboratory of the Army Air Service Engineering Division at Dayton, Ohio.

The torsional strength of a wing determines very largely the distribution of air forces upon it, and the tendency to flutter. Insufficient torsional strength produces wash-in or an increasing angle of attack toward the wing tips in the high incidence condition, further increasing the load on the front spar in the condition which is already the most severe. Conversely, torsional yielding in the low incidence and nose dive conditions produce washout of the wing shape and may exaggerate the critical condition for the rear spar.

The mathematical theory of the forces producing flutter is not yet sufficiently far advanced to determine by direct calculation the critical air speed at which flutter will commence. Comparison with successful practice must still be the principal criterion upon which to judge the adequacy of the torsional strength of a new design of wing. Obviously this comparison will be greatly facilitated by use of a coefficient of torsional rigidity including the principal factors in torsional strengths. A coefficient for comparing the torsional rigidity of different wings is derived in this report.

Report No. 331. Collection of Wind-Tunnel Data on Commonly used Wing Sections. By F. A. Louden. 45 pages, illustrations, quarto. Washington, Government Printing Office, 1929. Price, twenty cents.

This report was prepared at the request of the Committee in the Bureau of Aeronautics of the Navy Department in order to group in a uniform manner the aerodynamic properties of commonly used wing sections as determined from tests in various wind tunnels.

The data have been collected from reports of a number of laboratories. Where necessary, transformation has been made to the absolute system of coefficients and tunnel wall interference corrections have been applied. Tables and graphs present the data in the various forms useful to the engineer in the selection of a wing section.

Report No. 336. Tests of Large Airfoils in the Propeller Research Tunnel, Including Two with Corrugated Surfaces. By Donald H. Wood. 19 pages, illustrations, quarto. Washington, Government Printing Office, 1929. Price, ten cents.

This report gives the results of the tests of seven 2 by 12 foot airfoils (Clark Y, smooth and corrugated, Gottingen 398, N.A.C.A. M-6, and N.A.C.A. 84). The tests were made in the propeller Research Tunnel of the Committee at Reynolds Numbers up to 2,000,000. The Clark Y airfoil was tested with three degrees of surface smoothness.

The effect of small variations of smoothness of an airfoil is shown to be negligible. Corrugating the surface causes a flattening of the lift curve at the burble point and an increase in drag at small flying angles.

LEÇONS SUR L'HYDRODYNAMIQUE. Par, Henri Villat, Correspondant de l'-Academie des Sciences, Professeur a la Faculte des Sciences de Paris. v–296 pages, illustrations, 8vo, paper. Paris, Gauthier-Villars et Cie., 1929. Price 50 francs.

Extract from introduction: The substance of this volume is essentially the subject-matter of lectures at the Sorbonne which were assigned to me for the second semesters of 1925 and 1926 in the Course in Mechanics of Fluids.

In the first part, I have sought to give as complete an idea as possible, and as simple as possible, of the theory of wakes with respect to steady movement of a solid in a perfect fluid. This theory has been built up on the fundamental principles established by Kirchhoff, Helmholtz, M. Levi-Civita and M. Brillouin, and very considerable development, extension and refinement have been contributed by a number of authors in the first rank, among whom should be mentioned

M. U. Cisotti. I have myself, as well as my pupils (and notably M. R. Thiry), given some attention to the question. The development of these principles with an exposition of divers difficulties which arise from the theory, and the means employed for overcoming them, are examined; and stress is laid on the non-unique character of the solution, a fact which appears fundamental. The study of a perfect fluid regarded as a limiting case of a real fluid of definite viscosity may properly be made by considering how its characteristics and its motion are affected when the value of the viscosity approaches zero. That method has been employed by M. C. W. Oseen, and by his pupils, notably M. N. Zeilon.

The second part of this investigation is devoted to the consideration of this theory of M. Oseen. Passing to the limiting value of the viscosity as indicated introduces difficulties of considerable magnitude, as is to be foreseen from an examination by M. J. Boussinesq of a simple problem of this nature. As early as 1880 the latter had shown the non-analytical nature of a solution when the viscosity approached zero. We next proceed to the principles of the theory of Oseen which is based initially upon a transformation of hydrodynamic equations into integral equations, and finally after the method of M. Oseen, we arrive at a solution for the limiting case. The method thus employed gives one solution. We describe the results obtained by M. Zeilon in the research for the effective solution of important cases reduced to a form adapted to numerical calculation. We likewise describe the application of an extension of these theories to the case of fluids limited by fixed walls.

Our object has been to set forth theories due either to various authors or to ourselves as clearly as possible for a reader of adequate mathematical culture but who is not informed on the fundamentals of special problems. It is with a view to meeting the requirements of such a reader that we have introduced in this work a summary of elliptic functions in a form which permits an easy grasp of all the essential properties of these functions. Likewise a particular problem has served to set forth the useful properties of the best equation in a somewhat intuitive form, and for the solution of an example, we have given a brief account of the theory of the functions of Legendre which are of such importance in mathematical physics.

L. E. P.

THE DISPOSAL OF SEWAGE. A textbook for the Use of Engineers, Sanitary Inspectors and Students. By T. H. P. Veal, B.Sc. A.M.I.C.E., Lecturer in Civil Engineering at the University of Birmingham. XIV–173 pages, 48 illustrations, 26 tables, 8vo. New York, D. Van Nostrand Company, 1928. Price $4.25.

The author has made of this book an illuminating treatise on modern sewage disposal methods as practised in England. Primarily, his object has been to present the subject in a manner suitable for students or those who wish to acquire the main principles without unnecessary detail. In the accomplishment of this object he has been highly successful having arranged his subject matter with a perspective well delineated and thoroughly coördinated.

The book begins with a brief introductory chapter outlining the history of the development of sewage disposal methods especially as evidenced in a number of legislative acts. The first main topic deals with estimations on the quantity of sewage from any particular locality. Considerable quantities of data are given,

all of which serve to confirm the soundness of the calculations. The chapter on the analysis of sewage includes several typical sewage analyses and interpretations of the same. Under the caption of Preliminary Processes, various types of racks and screens are described that may be used to effect the removal of coarse, relatively heavy material which is apt to clog pumps or collect around the edges of the disposal basins. Several forms of settlement tanks, such as Vertical Flow, Travis, Imhoff and Precipitation tanks are described and methods given for calculating their required capacities. Considerable attention is paid to the subjects of bacteria beds and bio-aëration tanks as a means of treating the finely divided suspended matter. Construction of the beds and tanks, methods of application and distribution of sewage and their successful operation are outlined in a manner readily comprehended. A short chapter on land treatment is included. The success of this method depends upon a suitable type of soil being available in relatively large acreages. Such a requirement necessarily restricts a widespread adoption of this method. Several methods of sludge disposal are described and critically discussed. A chapter on Lay-out and Cost of Sewage Disposal Works deserves special mention. It includes descriptions of disposal works in a number of English towns and cities and concludes with estimates on the costs of construction and operation. A discussion of sewage disposal in rural districts proves as always, to be a timely subject.

The text is well supplied with illustrations and diagrams of machinery and tanks used in various sewage disposal works. Tables containing an abundance of data serve to supplement the numerous explanations and calculations.

T. K. CLEVELAND.

PUBLICATIONS RECEIVED.

The Fundamentals of Radio, by R. R. Ramsey, Ph.D., Professor of Physics, Indiana University. 372 pages, illustrations, 8vo. Bloomington, Indiana, Ramsay Publishing Company, 1929, price $3.50.

The Science of Voice. A book on the singing and speaking voice based upon the latest research in physics and physiology, with advice to those interested in talking movies and other mechanical producing devices by Douglas Stanley, M.S., A.C.G.I., with a preface by H. H. Sheldon, Ph.D. 327 pages, illustrations, 8vo. New York, Carl Fischer, Inc., 1929.

Introduction a la Mécanique de la Vie. I. Électrodynamique du Muscle par Charles Laville. 173 pages, illustrations, 8vo. Paris, Éditions Laville S. A., 1928, price 30 francs.

Diatomaceous Earth, by Robert Calvert. American Chemical Society Monograph Series. 251 pages, illustrations, 8vo. New York. The Chemical Catalog Company, 1930, price $5.00.

Metallurgy of White Metal Scrap and Residues, by Edmund Richard Thews. 383 pages, illustrations, 8vo. New York, D. Van Nostrand Company, Inc., 1930, price $5.50.

Asbestos. A list of references to material in the New York Public Library compiled by William B. Gamble, Chief, Science and Technology Division, 72 pages, 8vo. New York, The New York Public Library, 1929, price fifty cents.

Atlas Metallographicus, Hanemann und Schrader Lieferung 6, Tafel 41–48.

Lieferung 7, Tafel 49–56. 2 parts plates, quarto. Berlin, Gebrüder Borntraeger, 1929, price of each part, seven marks.

Union Géodésique et Géophysique Internationale. Section de Magnétisme et Électricité Terrestres. Comptes Rendus de l'Asemblée de Prague, Septembre 1927. Imprimés par les soins de Ch. Maurain, Secrétaire de la Section et Directeur du Bureau Central. 269 pages, map, 8vo. Paris, Les Presses Universitaires de France, 1929.

U. S. Department of Commerce, Coast and Geodetic Survey. Results of observations made at the United States Coast and Geodetic Survey Magnetic Observatory near Honolulu, Hawaii, in 1923 and 1924. 108 pages, quarto. Washington, Government Printing Office, 1929, price thirty cents.

National Advisory Committee for Aeronautics. Technical Notes No. 329; Some effects of air flow on the penetration and distribution of oil sprays by A. M. Rothrock and E. G. Beardsley. 11 pages, illustrations, quarto. Washington, Committee, 1929.

National Advisory Committee for Aeronautics. Fifteenth annual report for 1929. 89 pages, plate, quarto. Washington, Government Printing Office, 1929, price twenty-five cents.

CURRENT TOPICS.

Research Fellowships in Metallurgy and Mining. In coöperation with the U. S. Bureau of Mines and the State Mining Experiment Station, the School of Mines and Metallurgy of the University of Missouri offers four fellowships. These fellowships are open to graduates who have the equivalent of a Bachelor of Science degree and have had the proper training in mining, metallurgy, or chemistry, and who are qualified to undertake research work.

Fellows will register as students in the School of Mines and Metallurgy of the University of Missouri, and become candidates for the degree of Master of Science (unless this or an equivalent degree has been earned). Their class work will be directed by the heads of the departments of instruction, but about half of their time will be spent in research work under the direction of the Bureau of Mines staff resident at the School of Mines. The purpose of this work is to undertake the solution of definite problems confronting the mining and metallurgical industries of the State of Missouri. For 1930–31 the four fellowships will be granted in the following subjects: Ore Dressing; Problems in Gravity Concentration, Flotation, Ball Milling, and Magnetic Concentration.

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